God is also a mathematician.

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Queen's University
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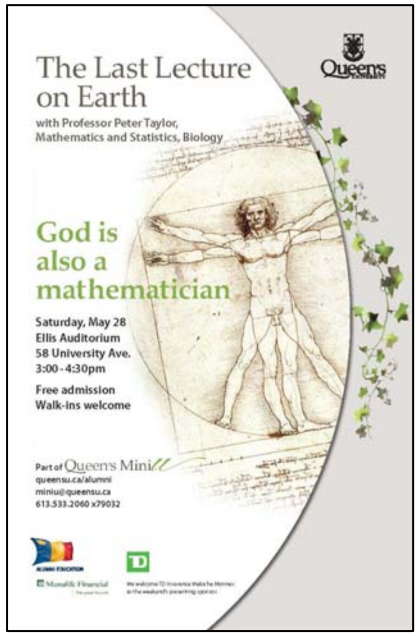
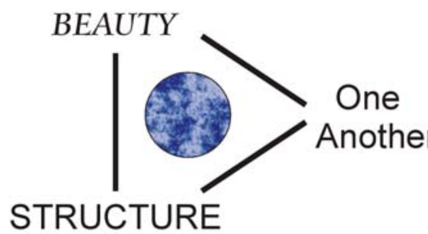


Mathematics and God have a long and deep relationship. Before the universe began, they are the only two things that existed, so they got quite used to whiling away the long dark night in one another's company, discussing irregular manifolds and improper integrals, and the universe that maybe one day they could build together. In this relationship, God was the artist and mathematics was the medium, out of which they were to construct the universe, and in that task they continually surprised one another with what they were able to do.



Let me declare my own holy trinity--the three ways that God reveals Himself to me, or if you like, the three faces of God. They are beauty, structure and one another and they are intimately related. They are also, of course, what drew me to mathematics. They are what I shall talk about this afternoon.

For me, this is a beautiful picture. It's called *Sunset at the North Pole*. Its beauty come from its structure. It is spare and quiet. It's familiar—we can make sense of it. Certainly the focus of the picture is that fantastical moon—it cries out to us, it excites and cradles us.

For sometime now the image has been wandering around cyberspace masquerading as a photograph, taken at a point at which the moon is closest to the earth. But this could not be. That moon could never be real, much as we might yearn it to be so. The orbits of the moon around the earth and the earth around the sun are almost circular and it happens that the sun and the moon always appear from the earth just about the same size. So the moon could never appear so much bigger than the sun. The picture is actually a work of computer art.

Why is mathematics so beautiful?

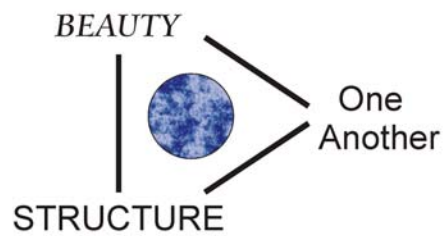
Why is the universe so beautiful?

Why is beauty such a powerful predictor of truth?

Why does mathematics have such power in unlocking the secrets of the universe?

In this talk I will give you the six problems that have absorbed me for most of my life. The first four are the problems of my youth—up until some 20 years ago.

The first of these, the beauty of mathematics, started in university to become transparent to me. Simply put—mathematics is the study of structure and beauty emerges from structure. Indeed my PhD thesis was titled "The structure space of a Choquet simplex" and from the topological structure of the set of extreme points of an infinite dimensional simplex, I could deduce properties of its internal geometric structure. For me it was a result of extraordinary beauty and from that time on, whenever I encountered objects of beauty, images, music, men and women, I would relate the experience to the underlying structure.



But it's not so easy to see how *the universe* fits in to all this? Choquet simplexes don't exist in the real world—they cannot be photographed. Why should beauty point to truth? And why should mathematics, whose existence seems not to depend at all on the universe, have such often unexpected success in describing it. Eugene Wigner referred to this as the "unreasonable effectiveness of mathematics." Physics has provided the most spectacular demonstration of this. We come up with a complex set of mathematical equations which predict some new weird particle, we build a billion-dollar accelerator, and lo we find it, exactly where it was predicted to be.

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And consider this crazy story. At time $t = 10^{-43}$, gravity separated out from the remaining three electronuclear forces, the universe was smaller than a quark and its temperature was 10^{27} degrees. Over the next 10^{-35} seconds, the universe expanded to be the size of a proton and the electronuclear force broke apart into its constituents: the strong nuclear, the weak nuclear, and electromagnetism. Then all hell broke loose—during the next 10^{-32} seconds the universe grew 10^{78} to be the size of a grapefruit.

It's not so much that the story is so crazy, though it is! *It's that we are able to tell it*. And with a straight face. And make no mistake it was the mathematics that took us there, that took us back 13.7 billion years in such unbelievable detail.

My answer to those last three questions about how the universe comes to be beautiful and structured and mathematical, is that the universe itself is structured. Note that there's no particular reason for that to be so. Perhaps properly structured universes are incredibly rare. Perhaps there are zillions of improperly structured universes out there—lacking that right delicate balance between order and chaos. Every artist has to practice and play and do crazy things. But life could never evolve in all those universes. The rare universes with mathematical structure will be the only ones able to harbour beings that are intelligent enough to wonder why their universe is structured.

An interesting recursion

We've been talking about mathematics. Some of you might have only a passing acquaintance with it—passing very fast perhaps! Perhaps it's time for an example.

1
3
4
2
- 4
- 12
- 16
- 8
16
48
64
32
⋮

Consider the sequence of integers at the right. It's a recursively defined sequence generated by the equation:

$$x_{n+1} = 2(x_n - x_{n-1}) \qquad \begin{cases} x_0 = 1 \\ x_1 = 3 \end{cases}$$

What this equation says is that we start with 1 and 3 and then each term is generated as twice the difference between the two previous terms. For example, to get the fifth term, calculate 2–4 = –2 and double that to get –4.

Now what we want is to get a better understanding of the structure of the sequence.

1
- 2
4
- 8
16
- 32
64
- 128
⋮

For example, this new sequence at the right has a much simpler structure—each term is easily seen to be –2 times the previous term with a sign change. Such a sequence in which each term is a fixed multiple of the previous term, is called *geometric*, and such sequences are taken to be the standard of simplicity and transparency in the sequence world.

The first sequence is not geometric, but if we study it closely we do see some geometric behaviour. If we consider successive blocks of four, we notice that each block is obtained from the previous block by multiplication by –4. It has a geometric block structure. That's certainly a great observation. The only obscure aspect remaining is the little dance the sequence executes within each block. How do we understand the sequence 1, 3, 4, 2?

1, 3, 4, 2
- 4, - 12, - 16, - 8 } x- 4
16, 48, 64, 32 } x- 4
⋮

Here's an idea. If the sequence really was geometric, if there really was a term-by-term multiplier *m* (which there isn't) what would it be? Well if it were applied four times in a row (to go from one block to the next) it would have to give –4. That is *m* would have to satisfy the equation $m^4 = -4$ and hence *m* would be a fourth root of –4:

$$m = \sqrt[4]{-4}$$

Well, –4 doesn't have a fourth root so maybe that's why there can't be a multiplier.

Any 1-term multiplier would have to be a 4th root of –4.

Actually, faced with such a situation, mathematicians in the past have simply invented what they needed, in this case a certain root of a negative number. The most powerful of such inventions has been i, the square root of -1.

When mathematician's invented i they referred to it as an "imaginary" number. Numbers of the form $ai+b$ are called *complex numbers.*

In fact, using i, we can manufacture a 4^{th} root of -4. It's simply $i+1$. The calculation appears at the right. We simply multiply using the normal rules, treating i as a symbol whose square is -1.

$$i = \sqrt{-1}$$

$$i^2 = -1$$

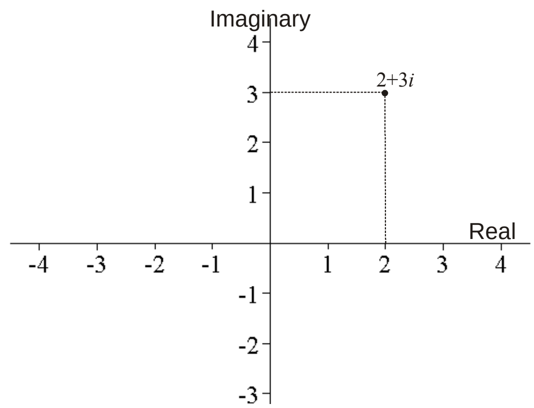
$$(i+1)^2 = i^2 + 2i + 1$$

$$= -1 + 2i + 1 = 2i$$

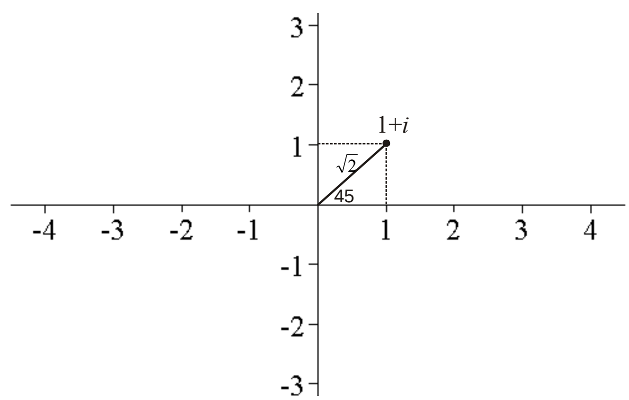
$$(i+1)^4 = (2i)^2 = 4i^2 = -4$$

So we have an imaginary candidate for our term-by-term multiplier m—it's $i+1$. But of course it isn't actually the multiplier of the sequence—for example to go from $x_1 = 3$ to $x_2 = 4$, you multiply by 4/3, not by $i+1$. So how do we make sense of this?

The breakthrough in understanding the structure of our sequence comes from the geometric representation of complex numbers. The number $a + ib$ is represented by the point (a, b) in the plane, with the x-axis regarded as the "real" axis and the y-axis regarded as the "imaginary" axis.



Now we will see how we can "see" that $1+i$ is a 4^{th} root of -4. First we locate it on the complex plane at the point with coordinates (1, 1). To multiply by this number we have to know its distance from the origin ($\sqrt{2}$ --called its *modulus*) and the angle it makes with the x-axis (45º--called its *argument*).



It turns out that there's a wonderfully geometric way to multiply two complex numbers. If z_1 and z_2 are two complex numbers with moduli r_1 and r_2 and arguments θ_1 and θ_2, then their product z has modulus the product

$r = r_1 r_2$

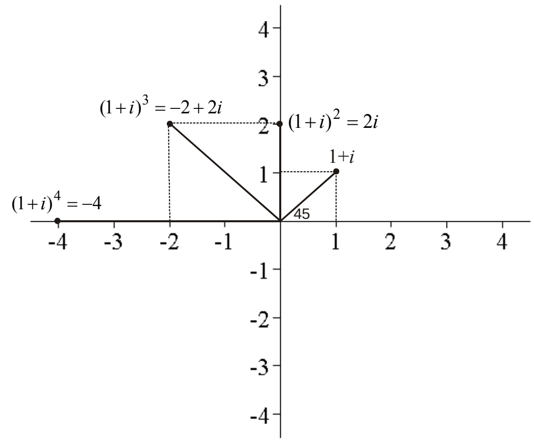
and argument the sum

$\theta = \theta_1 + \theta_2$.

That is, you multiply the moduli and add the arguments!

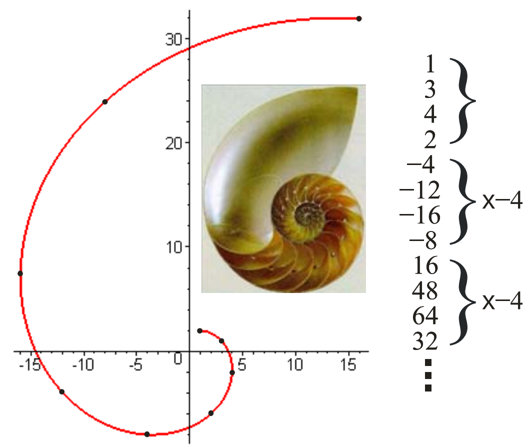
What that tells us is to calculate successive powers of a complex number we keep multiplying by the modulus and adding the argument. Thus, if z has modulus r and argument θ, then z^n has modulus r^n and argument $n\theta$.

This principle allows us to easily display the powers of 1+*i*—we simply keep rotating through 45º and multiplying the distance from the origin by $\sqrt{2}$. This is illustrated in the graph at the right, and we clearly see that $(1+i)^4 = -4$.



Now we need a reward for all that work with "imaginary" numbers. Remember we are after some way of understanding how 1+*i* might be a multiplier for that sequence.

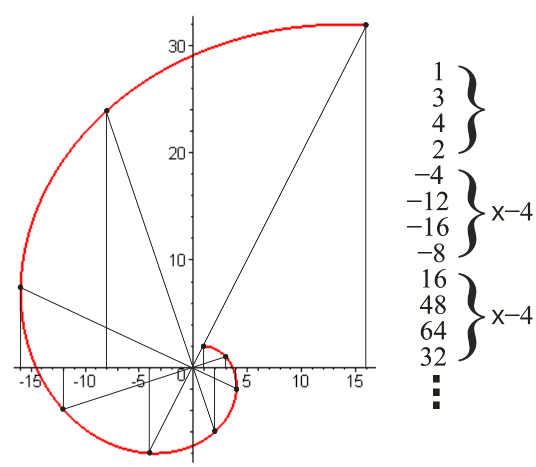
What we do, essentially, is lift the sequence into complex number space, that is, we give it an imaginary dimension, and see how it then behaves.



And the result is stunning. At the right, we see the sequence in complex number space. The terms (the points) are seen to lie on a graceful spiral. It's called a logarithmic spiral and is exactly the form employed by sea-shells. The same mathematics that drives the sequence also powers the living world.

As a nice exploration can you see why it's still true that $z_{n+1} = 2(z_n - z_{n-1})$?
For that you need to know that complex numbers add like vectors (*a*, *b*). For example, to show that $z_3 = 2(z_2 - z_1)$, draw an arrow from z_1 to z_2, double its length, and translate the new arrow so its tail is at the origin. Its head should be at z_3.

At the right we see the anatomy of the complex sequence. The vertical lines project the points down to their real parts, 1, 3, 4, 2, –4, etc. and the radial lines display the rotation by 45º. [Actually the rotation is through –45º. That's because in the complex representtation I have chosen, the points are obtained from one another through multiplication by 1–*i* rather than 1+*i*. Note that but 1–*i* is also a 4th root of –4. In fact –4 has four 4th roots. Can you find the other two?]

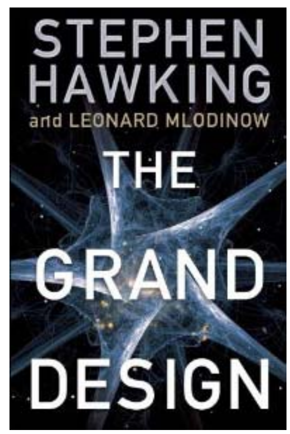


This example illustrates the capacity of mathematics to reach into an imaginary world and thereby give us a more powerful understanding of the structure of its constructs, and ultimately, of the structure of the real world itself.

A glimpse of God

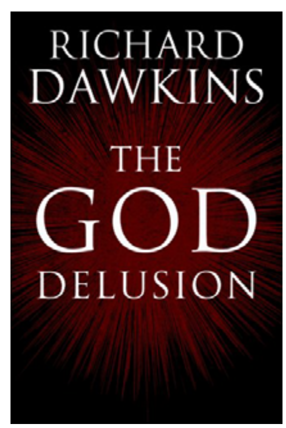
That's a glimpse of mathematics. Now for a glimpse of God. He's the chap on the right.

Long ago we kept him pretty busy orchestrating all the comings and goings in a wild and wooly universe. But with the rise of science, we found he was needed less and less. In Physics, Copernicus, Archimedes, Galileo, Newton, Einstein, Heisenberg successively banished him to a distant corner. It is with some relief that we understood that he was at least required to start things off, because although physics can get crazily close to time zero, as we have seen, it can't go all the way.



But Stephen Hawking now claims that Physics can even do that, though, rather inelegantly, I thought, with M-theory. The M in M-theory stands for "multi" because you have to stitch a whole bunch of partial theories together. Not, I might say, what we've come to expect from Physics. For the same reason, today's kids have been called the M-generation because they're a pastiche of electronic gadgets and social networking tools which is why they have a hard time focusing on any one thing. I'm sure you all know what I mean.

Evolutionary biology has also done a good job of pushing God out of the picture. Darwin showed that God wasn't really needed to explain the richness of the living world, and then Huxley, trounced the good Bishop Wilberforce in managing the jump from ape to man. Finally Dawkins assures us that evolution can even account for religious experience and our invention of the soul.



I remark that although I started my scientific life with Physics, the rapidly expanding power of mathematics to do so much with the living world is what prompted me, 35 years ago, to switch to theoretical biology.

God, must be quite amused and I think quite grateful, to see the evidence for his existence whittled away by the advances of physics and evolutionary biology, that is, by his old friend mathematics. He was never so keen on being regarded as the super in the basement keeping the furnace going, even when it's such a grand furnace. He has better things to do.

Jokes

I was telling a friend about this talk and after he listened a bit he said: so how many jokes are you going to tell?

Jokes? I don't do jokes.
Hmm. Is it afternoon or evening?
Afternoon. Three o'clock.
Three jokes. You need three jokes.

He turned to go and them turned back:
And for the love of God… I perked up. This was clearly relevant.
For the love of God, Peter, don't be so damn serious.

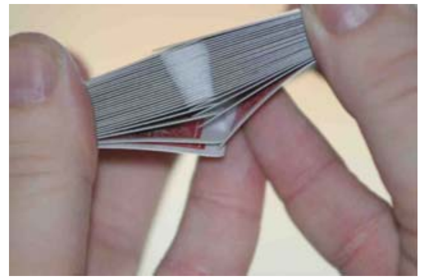
Okay. I have three jokes. They're not mine and you might have heard some of them before but they are all worth retelling. Their common theme is *Things math is not supposed to do*.

The first joke.

http://www.youtube.com/watch_popup?v=h60r2HPsiuM&feature=youtube_gdata_player

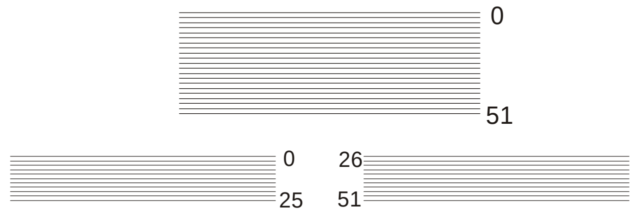
I like this joke. Every time I watch it I smile and nod my head.

The second joke.



The hands you see are those of Computer scientist Brent Morris doing a "perfect shuffle" He has a PhD in card shuffling from the Math Dept at Duke University. It turns out that his card-shuffling algorithms have great applications to data manipulation in computer science.

But they also allow him to do an amazing card trick. He gives you the deck and asks you to choose a card, put it on top and give the deck back. He then asks what position you would like your card to have in the deck. Suppose you say 46 (the 46th card from the top). He then (perfectly) shuffles the deck a few times (in this case 6 times) and counts from the top. Your card is indeed the 46th card counted.



Quite spectacular.

Let’s see how it works. First we number the cards in the deck from 0 to 51. [Because of the special status of the top card, it needs to have the number zero. So that when you say you want your card to be the 46th card, that’s actually card #45 for him.]

OUT Shuffle: Top card goes out	0
	26
	1
	27
	2
	⋮

Now what’s a perfect shuffle? The deck is divided into two equal halves and riff-shuffled so that the cards fall alternately. However, there are two ways to do that depending on which side releases the first card. In the two cases, the top card of the deck either remains on top (an OUT-shuffle), or it gets tucked in to the second position (an IN-shuffle).

IN Shuffle: Top card goes in	26
	0
	27
	1
	28
	⋮

Okay—It turns out that with the right sequence of IN and OUT shuffles, we can move card #0 into any position we want, for example to be card #45. Start by writing 45 as a sum of powers of two. Do this by taking out the largest power 32, leaving us with 13, take 8 out of that, leaving 5 which is 4 plus 1. Now we need to keep track, not only of the powers we used, but also of the ones we didn’t, like 16 and 2, and indeed we write the sequence of multipliers, 1, 0, 1, 1, 0, 1 that identifies this sequence. In fact, 101101 is the base 2 representation of 45 (for those who recall that wonderful chapter of your elementary school life).

32
8
4
1
45

1x32
0x16
1x8
1x4
0x2
1x1
45

Finally it’s time for the punch line (are you ready to laugh?). Take that sequence 101101 and do a little surgery on the 1 and fatten the 0 a bit and get IOIIOI and that gives you the sequence of IN and OUT shuffles which moves card#0 to card#45.

1 → I → IN
0 → O → OUT
1 → I → IN
1 → I → IN
0 → O → OUT
1 → I → IN
45

Crazy.

Let me turn to my last two questions—both currently of great interest to me.

Where does the sense of structure in our lives come from?

Why is there so much conflict in human affairs?

Where does the sense of structure in our lives come from?

There is no God: the revelation came to Dan Kellogg in the instant he saw the World Trade Centre South Tower fall.

So begins one of the stories in John Updike's last collection: *My Father's Tears*.

From the very beginning of our lives we try to make sense-- to find meaning in what happens. We all sense a governing structure in our lives and we seek to understand it and divine its meaning and when things go terribly wrong we feel brutally betrayed. Yet we keep looking. Because we feel deep down that there is a meaningful web that knits the moments of our experience together.

This is a time-honoured dilemma. In the late 1st century AD, Tacitus in his *Annals* wrote:

I am undecided whether the affairs of human beings evolve by fate, and an immutable inevitability, or by chance.

Tacitus

Fate is the unseen hand; chance is the roll of the dice.

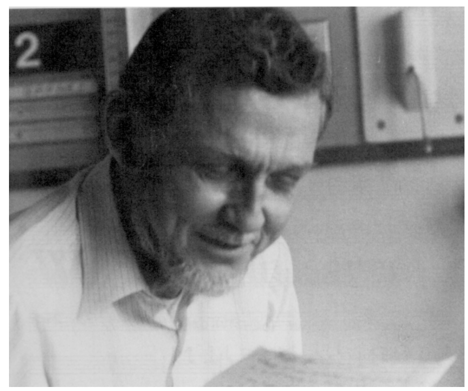
My fascination with structure is about understanding how there can be a plan, a design, yet not laid out beforehand, so that I am not only the spectator, but also the architect, of my own journey.

We have all had experiences like this. You enter a room and you know that your life is about change forever. And then you go around a corner…

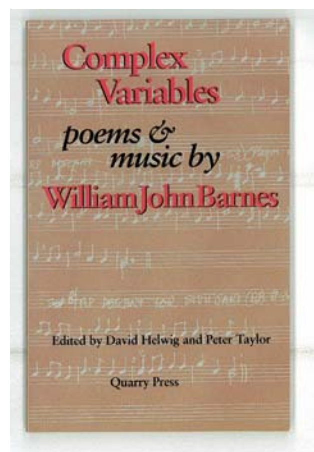
Or this. You get home from work, running a bit late and there's a couple of students coming for dinner in an hour, and you haven't quite decided what to cook. And then for no apparent reason, you jump back on your bike and ride to the house of a valued colleague whom you haven't run into for a couple of months. He's in and in fact his three children are there from out of town and they are all about to go out for dinner. He's in good shape and introduces you to his children. And then off they go for their dinner and you speed home for yours. The next night you meet his children again at KGH, one having returned to Toronto that morning and then just now rushed back to Kingston. Your colleague never regains consciousness.

How are we to make sense of such experiences?

My colleague in that story is actually Bill Barnes, for many years my wonderful co-teacher in our Math and Poetry Course. Perhaps some of you were privileged to have Bill as a teacher, or as organist and choir-master at St. James. For most of his life he struggled with the ravages of diabetes, in and out of the hospital, and much of his poetry drew from that experience.



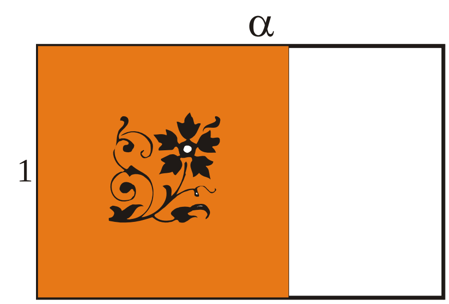
After he died, David Helwig and I collected his poems and music. The titles was actually Bill's idea, one day musing what he would call his poems, collectively. I like how it fits with the complex numbers we worked with in our spiral recursion. I saw it archived once on Amazon in the mathematics section. I have a few copies of this book left which are free for the taking. Just ask.



Before I go on to the last question, it is perhaps time for the last joke.

The third joke.
This is about two characters whom I shall call alpha and beta, though they also go by other names. No, they didn't meet in a bar, but if they had so met they'd find they had some unexpected things in common. It turns out that each of these numbers is associated with a special shape of rectangle.

Alpha. A rectangle with height 1 and width α has the property that if you put a square carpet at one end, the uncarpeted area has the same shape as the whole. That means that the two rectangles have the same length/width ratio, and if we set these equal, we get



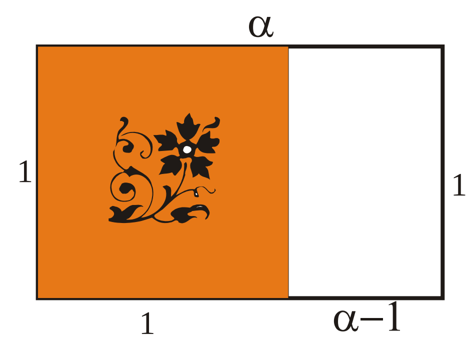
Golden Rectangle

$$\frac{\alpha}{1} = \frac{1}{1-\alpha}$$

And this can be rearranged into the quadratic equation

$$\alpha^2 - \alpha - 1 = 0$$

which solves to give:



$$\alpha = \frac{1+\sqrt{5}}{2} \approx 1.618\,.$$

Alpha is called the golden mean, and the $1\times\alpha$ rectangle is called the *golden rectangle*. Such rectangles are supposed to provide the most aesthetically pleasing proportions and they are often found in Greek architecture.

Salvador Dali's *Last Supper*. is framed in a golden rectangle.

Beta. A rectangle with height 1 and width β has the property that if you carpet exactly half of it, the uncarpeted area has the same shape as the whole. That means that the two rectangles have the same length/width ratio, and if we set these equal, we get

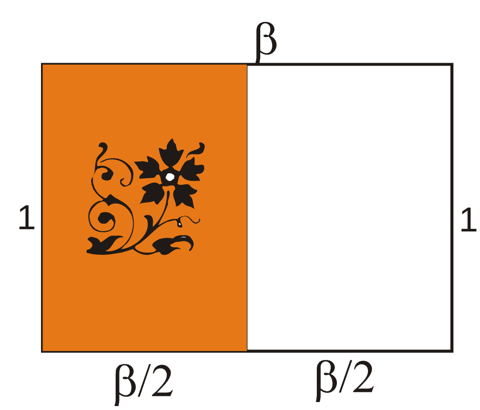
$$\frac{\beta}{1} = \frac{1}{\beta/2}$$

And this can be rearranged into the quadratic equation

$$\beta^2 = 2$$

which solves to give:

$$\beta = \sqrt{2} \approx 1.414\,.$$



Thus, beta is simply the square root of 2.

Perhaps the β-rectangle is not so aesthetically pleasing as the α-rectangle, but it is deemed functional by some as it is the shape of the standard A4 paper in Britain.

Now let's see what happens when these two fine friends wander out to play games together.

Here we have two rows of four tables, the α-tables above and the β-tables below. Let's start with the α-tables. The first table lists the multiples of α—first α, then 2α, then 3α, etc. Lots of ugly decimals, and of course there are lots more (!) that we didn't provide.

All those decimals remind me of what my basement looked like 10 years after all my kids have left home and even got their new homes and why on earth do I still have all this "stuff"? So one day (after repeated warnings!) I decided to simply get rid of it. It was a great feeling but it sure was scary. What if I threw away something important?

That's what we did for the second table—we simply threw all those messy decimals away, leaving us with nice clean integers. Mathematically, that's a totally crazy thing to do. We didn't even round up or down, we just pitched!

In the third table we noticed that no all integers appeared in the second table, so we simply listed the ones that were missing. That's columns 3. And then in column 4 we subtracted column 2 from column 3—something else that makes no mathematical sense.

But look what we got—the integers in order! Weird. How could that have happened?

But there's more crazy stuff to come! When we do the same thing to β, we get not the integers in order, but their doubles! Double weird. What on earth is going on here?

I've called this a joke and when I first saw it I certainly laughed and laughed. I hope you have as well.

n	***nα***
1	1.618034
2	3.236068
3	4.854102
4	6.472136
5	8.09017
6	9.708204
7	11.32624
8	12.94427
9	14.56231
10	16.18034
11	17.79837
12	19.41641
13	21.03444
14	22.65248
15	24.27051
16	25.88854
17	27.50658
18	29.12461
19	30.74265
20	32.36068
21	33.97871
22	35.59675
23	37.21478
24	38.83282
25	40.45085

n	***nα***
1	1
2	3
3	4
4	6
5	8
6	9
7	11
8	12
9	14
10	16
11	17
12	19
13	21
14	22
15	24
16	25
17	27
18	29
19	30
20	32
21	33
22	35
23	37
24	38
25	40

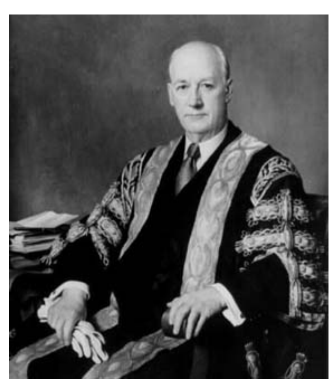
n	***nα***		
1	1	2	**1**
2	3	5	**2**
3	4	7	**3**
4	6	10	**4**
5	8	13	**5**
6	9	15	**6**
7	11	18	**7**
8	12	20	**8**
9	14	23	**9**
10	16	26	**10**
11	17	28	**11**
12	19	31	**12**
13	21	34	**13**
14	22	36	**14**
15	24	39	**15**
16	25		
17	27		
18	29		
19	30		
20	32		
21	33		
22	35		
23	37		
24	38		
25	40		

n	***nβ***
1	1.414214
2	2.828427
3	4.242641
4	5.656854
5	7.071068
6	8.485281
7	9.899495
8	11.31370
9	12.72792
10	14.14213
11	15.55634
12	16.97056
13	18.38477
14	19.79899
15	21.21320
16	22.62741
17	24.04163
18	25.45584
19	26.87005
20	28.28427
21	29.69848
22	31.11269
23	32.52691
24	33.94112
25	35.35533
26	36.76955
27	38.18376

n	***nβ***
1	1
2	2
3	4
4	5
5	7
6	8
7	9
8	11
9	12
10	14
11	15
12	16
13	18
14	19
15	21
16	22
17	24
18	25
19	26
20	28
21	29
22	31
23	32
24	33
25	35
	36
	38

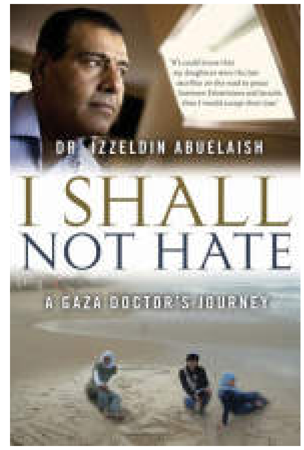
n	***nβ***		
1	1	2	**2**
2	3	5	**4**
3	4	7	**6**
4	6	10	**8**
5	8	13	**10**
6	9	15	**12**
7	11	18	**14**
8	12	20	**16**
9	14	23	**18**
10	16	26	**20**
11	17	28	**22**
12	19	31	
13	21	34	
14	22	36	
15	24	39	
16	25		
17	27		
18	29		
19	30		
20	32		
21	33		
22	35		
23	37		
24	38		
25	40		

Not surprisingly there's a deep and beautiful theorem behind this phenomenon called Beatty's Theorem. Interestingly, Samuel Beatty was a math professor at University of Toronto and was her Chancellor from 1953 to 1959.



Why is there so much conflict in human affairs?
I guess the first answer is that we're simply products of our evolutionary past.

But we've slipped the bonds of evolution in both directions, first in being generous and loving and secondly in being cruel and resentful. Much of my own research work concerns the evolution of conflict and cooperation. I haven't yet done any work on *human* behaviour, but I hope to start a project this fall with a new post-doc related to competition and collaboration (actually I prefer the word "community") in undergraduate learning.



We are intelligent and we can all understand the tragedy of the commons, we can all clearly see that generosity and love is a better way for society, indeed for the world. We are also spiritual and we are able to feel within ourselves the pain of others. And there's lots of pain out there. It's huge.

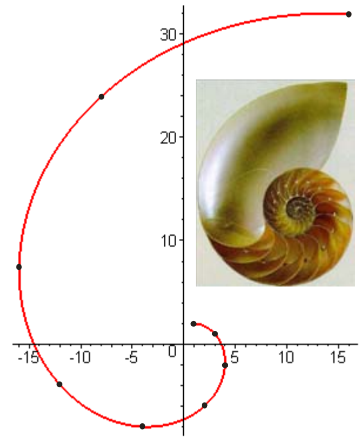
But somehow we can't seem to get to where we really want to be, to where we know we really ought to be. I have just read a remarkable book on this theme: *I shall not hate.*

I have learned a lot about conflicting desires from my cousin **Candasiri**, my Father's niece, who is a senior nun in the Theravada Buddhist tradition at the Amaravati Monastery in Hertfordshire.

In her former life, Candasiri found that she was always struggling with desire—between following or repressing them. There seemed a perpetual war inside of her. She talked about this in a BBC broadcast she made in 2008 on Jesus as seen through Buddhist eyes. The Christian way, the teachings of Jesus, seemed to require that she resolve this conflict, either to surrender or to overcome, depending on the measure of her strength and will. Buddhism seemed to offer her another way, to just let it go. Through meditation she found she could simply bear witness to these desires, and allow them to pass on according to their nature. For her this involved stepping away from the world as she had known and lived it.

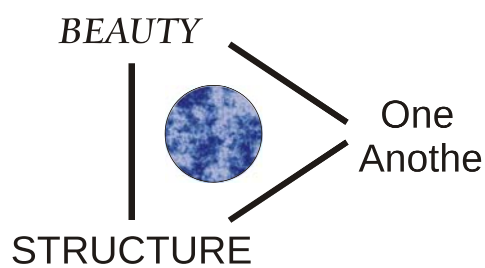
I know that the monks and nuns in Candasiri's tradition has been criticized for being so helpless, for its total dependence on the generosity of others for its very sustenance. At first glance, her way of life seems unworkable, unreal.

Candasiri’s way is certainly not for most of us, it might even work against our nature. That oversize moon is not part of our nature either, nor is the 4th root of –4, but they both give us beauty and a new way of understanding. Candasiri gives me that too—spare but larger than life, even unreal, but reflecting the sun, and echoing the sea-shell, not brightly, but quietly, gently, in a simple elegant curve.



Time to close. I return to the realization that this is my last lecture. Thank you for being here. Life can be lonely and so can death. We keep ourselves busy, our schedules full, so much to do. When we happen to cross paths with a neighbour or an acquaintance we say hi and carry on. That’s mostly how we live. It’s good. I am blessed to have good work, good students, good, if occasionally crazy, colleagues. I would miss lecturing—I enjoy it. You get to gather your thoughts, decide just how to organize them, honour their beauty, their fine structure, and then you get an entire class, some quite enthusiastic, to listen and respond.

For me, doing mathematics is quite analogous to being with God. They are both out there, just within reach, though you are never sure whether you are dealing with reality or with an elegant mythological world that we have inexplicably been given access to. They both do beautifully structured work which we can enjoy together. To be able to serve them both gives me a sense of wonder and humility.



As my father drew towards the end of his life he became gentle, so very different from his way when he was young, quite fierce really. So what do I say at the end?––Be gentle. Do mathematics, spend time with your God. Be joyful too. Some people suffer enormous burdens in their lives. I am sometimes overwhelmed at the realization. Be gentle. Thank you.