Letters to the Editor

Why the Sky Is Still Falling

In an article in the January 2009 *Notices*, David Bressoud is alarmed by the declining enrollment in math departments and obtains much of his data from enrollments in calculus courses at several levels. In North America, more students than ever take calculus in high school but fewer than ever take advanced calculus in university. This is significant because, as Bressoud observes, "calculus is at the heart of the mathematics curriculum" (p 22). If we want to understand this pattern of declining interest in mathematics, our view is that we need look no further than that statement.

We are convinced that the increasing adoption of calculus as the ultimate course in the high school curriculum has had a significant negative effect on mathematics enrollments in university. The reason is simple. Students learn (and learn how to learn!) by "playing"—reinventing, reconstructing—and calculus is not so easy to play with, nor is it particularly enticing for most students. Certainly students in high school are not ready and able to play with calculus, except possibly in the hands of an exceptional teacher. As Bressoud's data confirms, the introduction to higher mathematics provided by calculus convinces most of them that it's an important and sophisticated subject, but not for them.

Our view is that the final high school course should work with problems and investigations that bring together and build on the methods of algebra, geometry, and functions that have been developed in the earlier grades and open the way to a number of more sophisticated ideas from, for example, probability, recursive thinking, stability, invariance, transformation, and mathematical analysis. The problems that we use to do this should draw them in and challenge them. Given that, they will spend the time playing with their analysis, both working with one another and inventing their own solutions, and they will finally gain that crucial sense of mastery that one needs to move confidently forward in the subject.

As things stand, the final high school course is not at all like that. Why is that? Where does the responsibility lie for the growing emphasis on high school calculus? In our view it is the fact that calculus is the default mathematics service course for students in science, business, health, and several other areas, in their freshman year at college or university. It is also the default filter for future teachers of mathematics—and that generates a damaging feedback loop into high school mathematics classrooms. A good part of the reason for this is historical. There was a brief movement in the early 1980s, anticipating the advent of universal computer power, that promoted algebra and discrete mathematics. But a vigorous response in the late 1980s and early 1990s which emphasized the beauty and historical importance of mathematical analysis led to the highly successful calculus reform movement. And it has to be said that this movement carried with it new ideas of mathematical modeling and investigative approaches to problem solving. However, in a context focused on completing calculus early, and with teachers filtered through calculus as "the heart of the mathematics curriculum", the content lists continue to dominate the promising possibilities of reform teaching.

Now, twenty years later, maybe it's time to look again at what we teach when and why. And perhaps this time there are two questions that we might bear down upon:

(1) student engagement—what types of problems will entice the student into a sustained active learning mode?

(2) professional needs—what mathematics does the student need after graduation, and moving through the profession or the workplace?

We feel that the second question alone would lead to a mathematics program, at both the school and university level, that was much more algebraic, geometric, and discrete, emphasizing a number of the big ideas above, rather then the systematic coverage of calculus and differential equations we see at the moment. At the university level, this applies particularly to our extremely important "service" courses for students from a wide range of disciplines such as biology, psychology, economics. For these students we would like to see a course that is more collaborative, literate, and problems-based. We favor an approach that looks for the world in a grain of sand, and in the lively exchanges we would have with our students.

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Submitting Letters to the Editor

The *Notices* invites readers to submit letters and opinion pieces on topics related to mathematics. Electronic submissions are preferred (notices-letters@ams.org); see the masthead for postal mail addresses. Opinion pieces are usually one printed page in length (about 800 words). Letters are normally less than one page long, and shorter letters are preferred.