The aesthetics of mathematical modeling for the classroom

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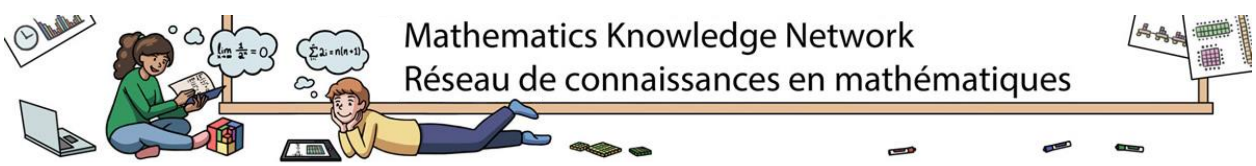


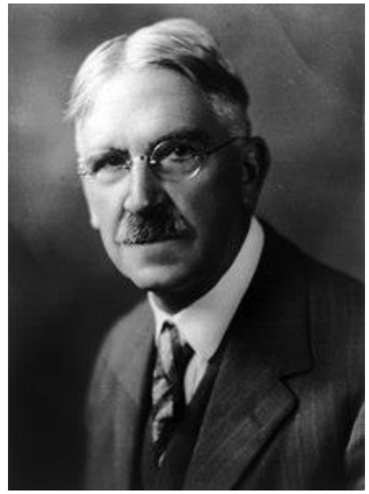
A serious problem

We’re letting all our students down—and in the same way.

Teachers too

Curriculum is a laundry list—it is not based in meaningful activities, activities that are central to their lives.

Huge research telling us what we need to do



Alfred North Whitehead

1861-1947

The Aims of Education (1922)

John Dewey

1859-1952

Art as Experience (1934)

Seymour Papert

1928-2016

The Unconscious mind (1978)

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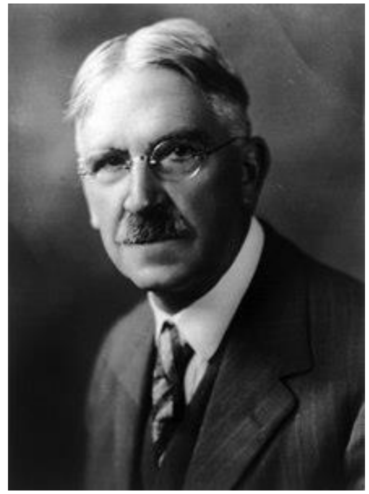

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What does it mean to be human?

Homo Aesthetheticus

Aesthetics and beauty.

Homo Aestheticus

Aesthetics and beauty.

Music, Drama.

So where do we look for our subject matter, our curriculum?

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Look to mathematics
--to what mathematicians do.

And what is that?
What is mathematics?

Structure

Mathematics is the abstract study of structure.

Structure

Mathematics is the abstract study of structure.

A huge part of this is design.

It is in the act of designing
that we begin to serious grapple with structure.

This is a huge challenge for the secondary system

teachers

students

The high school mathematics laundry list

Our students do not need it.
Neither does the world.
Those *few* who do need it will master it—
––because they need it and they love it.

Implications for university and college

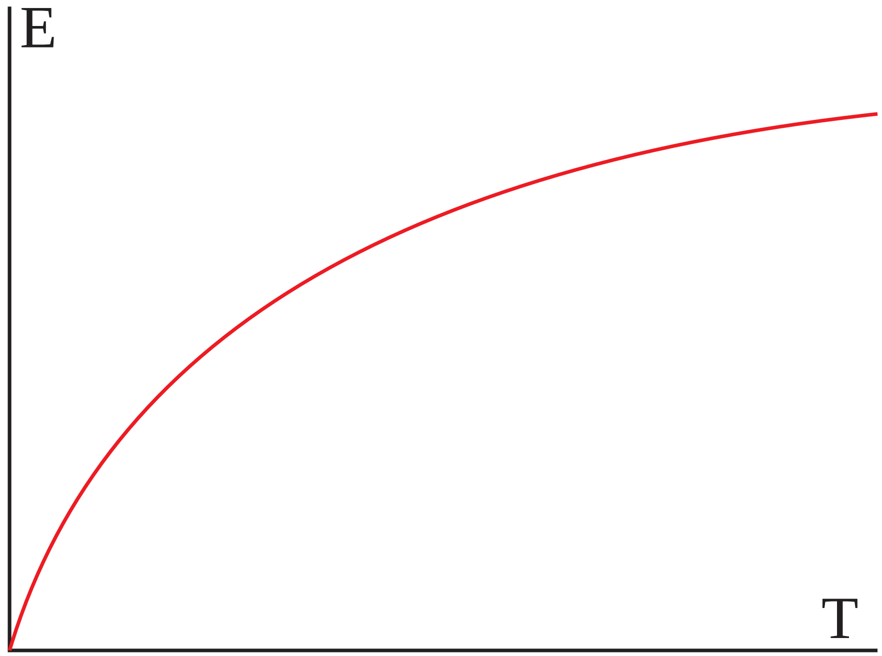
This approach to secondary curriculum poses a huge challenge to the tertiary level.

In fact, the universities have lost their way.

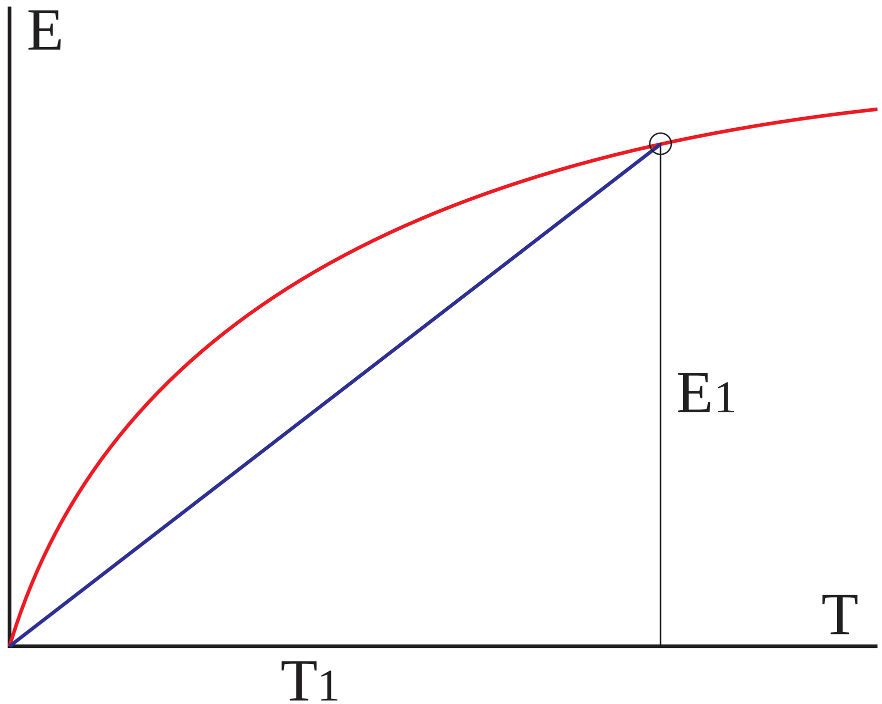
Optimal foraging

Sophisticated aesthetics for the Grade 12 student.

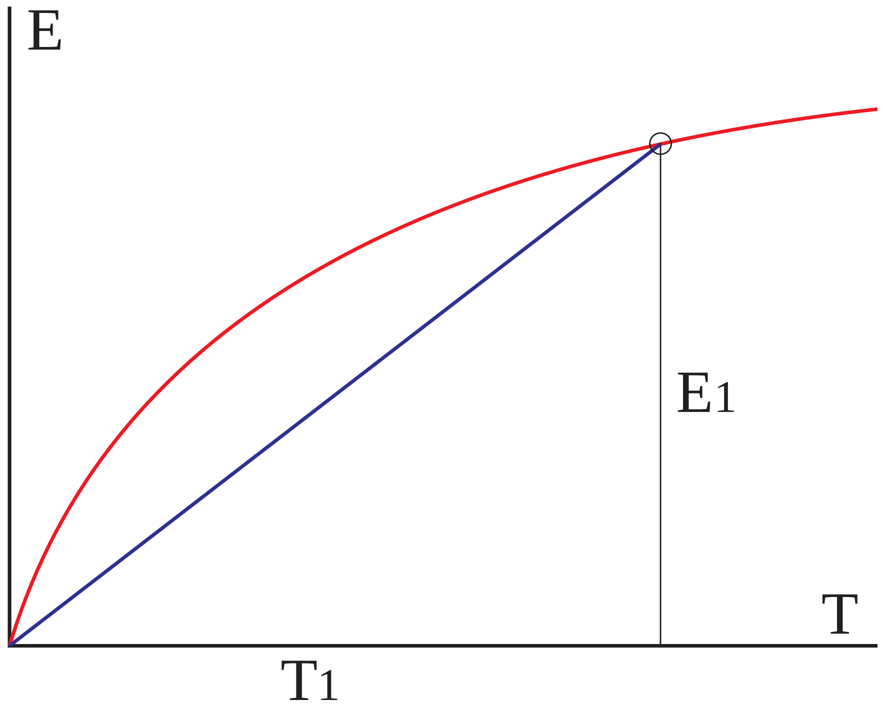


Calories gained E against time spent T foraging in patch

We want to maximize calorie gain per unit time.



calorie gain per unit time $= \dfrac{E_1}{T_1} =$ slope of line



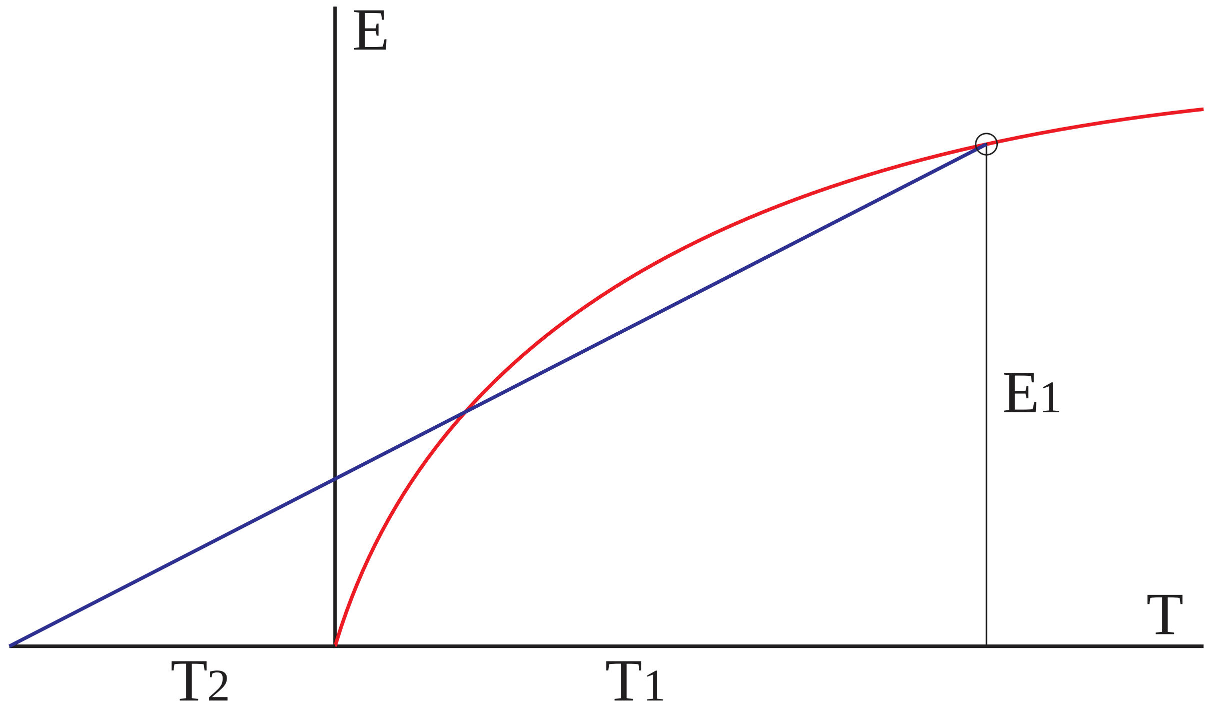
calorie gain per unit time $= \dfrac{E_1}{T_1} =$ slope of line

But must account for time T2 spent searching for fresh patch

Foraging time	T1
Searching time	T2
Cycle length	T1 + T2
Calories gained	E1

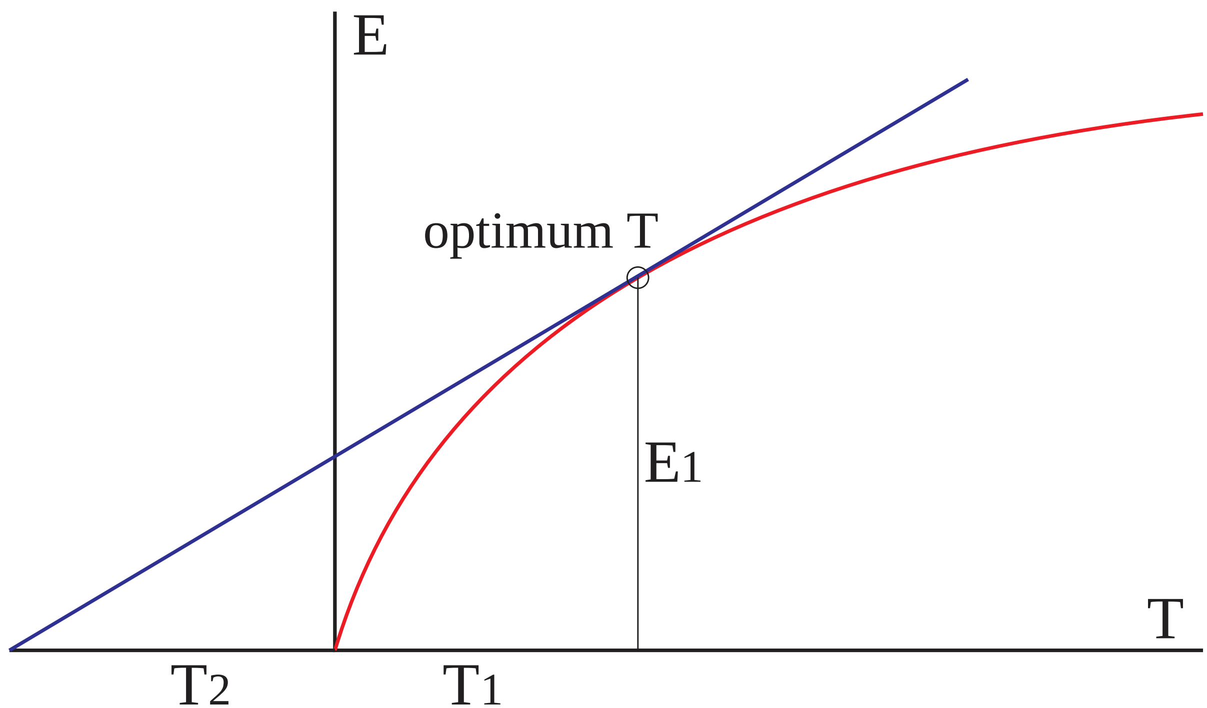
$$\frac{\text{calories}}{\text{cycle}} = \frac{\text{E1}}{\text{T1} + \text{T2}}$$

How do we maximize this?



Bird spends time T2 searching for a fresh patch

$$\frac{\text{calories}}{\text{cycle}} = \frac{E1}{T1 + T2} = \text{slope of line}$$



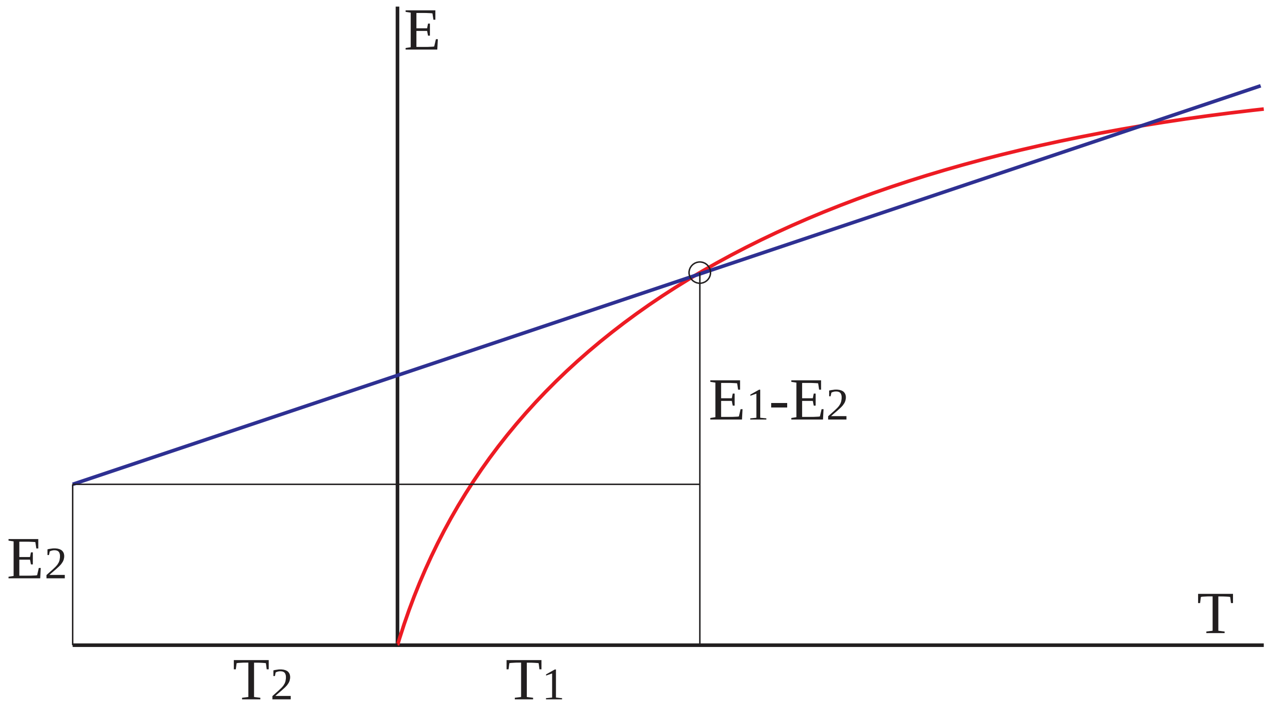
Slope is maximized by the tangent line

While the bird is flying around searching it is burning up some of those huckleberries that it just ate.

Foraging time	T1
Searching time	T2
Cycle length	T1 + T2
Calories gained foraging	E1
Calories lost flying	E2
Net calories	E1 – E2

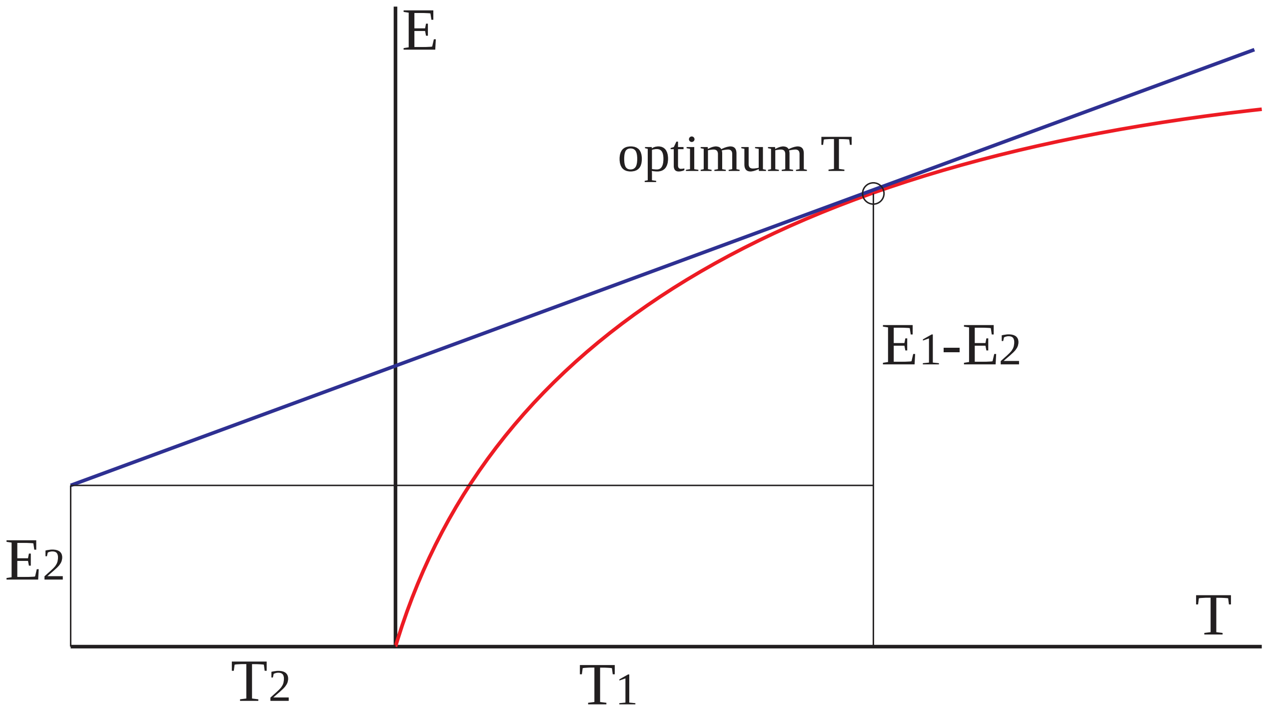
$$\frac{\text{calories}}{\text{cycle}} = \frac{\text{E1} - \text{E2}}{\text{T1} + \text{T2}}$$

How do we maximize *this*?

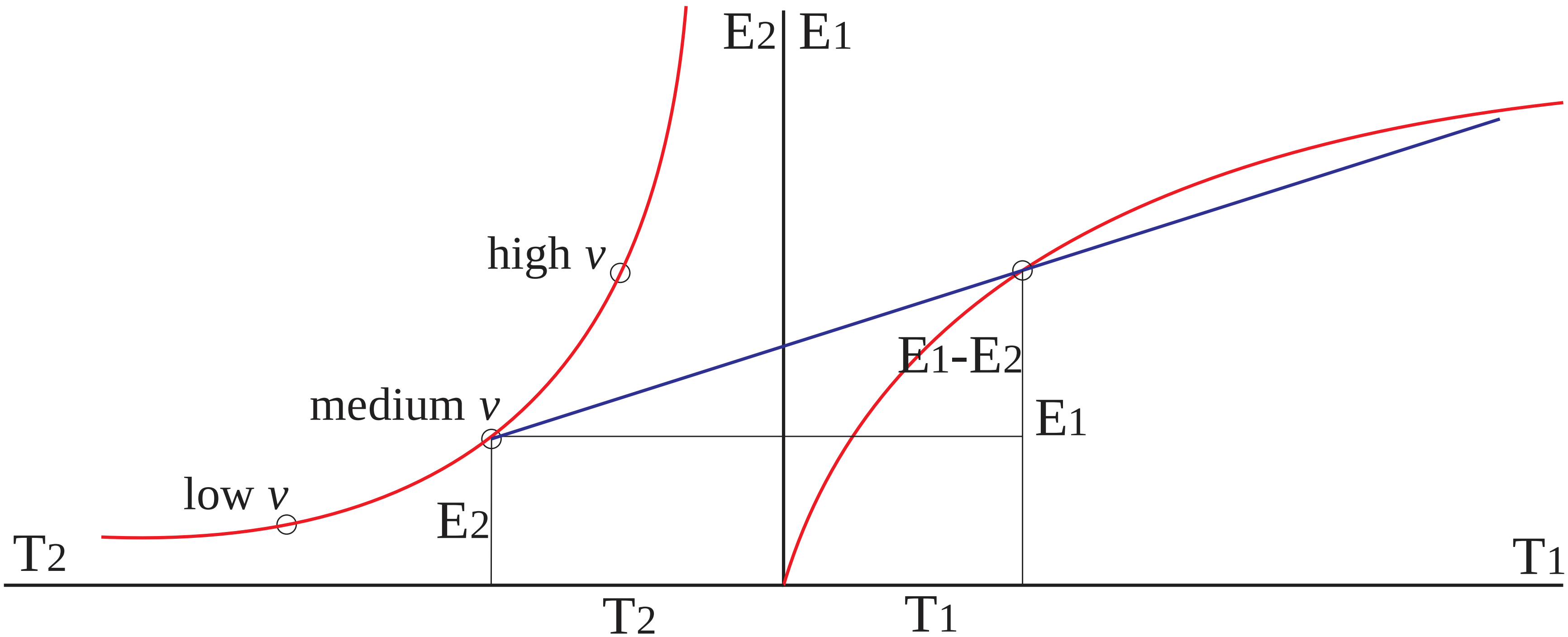


Bird spends time T2 searching for a fresh patch losing E2 calories

$$\frac{\text{net calories}}{\text{cycle}} = \frac{E1 - E2}{T1 + T2} = \text{slope of line}$$

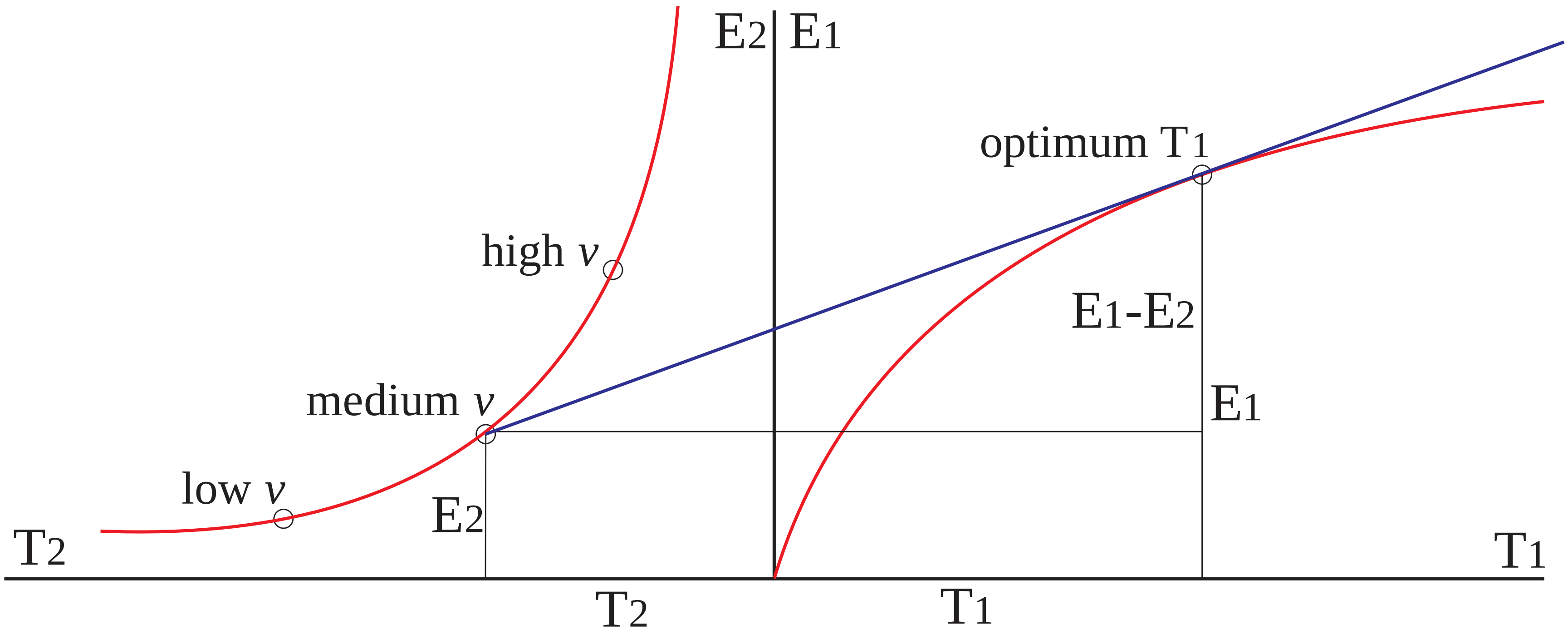


Slope is again maximized by the tangent line

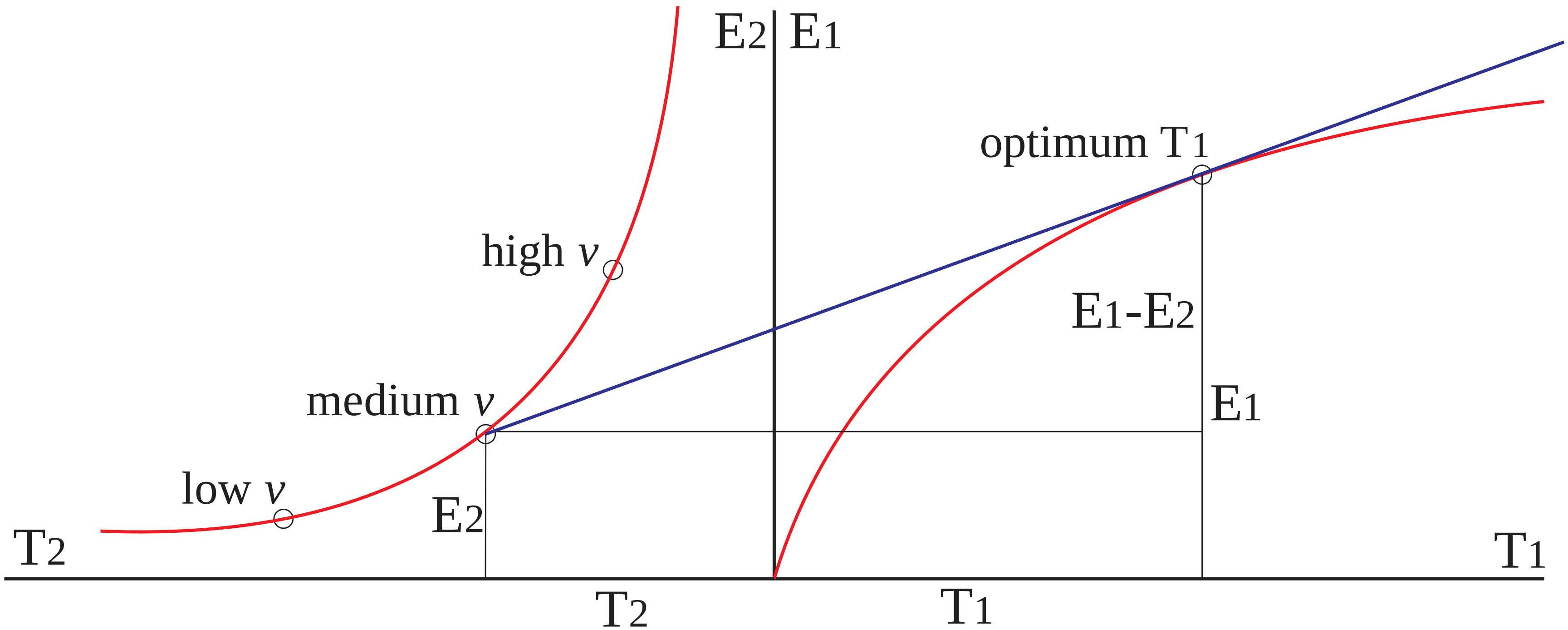


Now the bird has two decisions:
how long to forage and how fast to fly.

We have a two variable optimization problem.

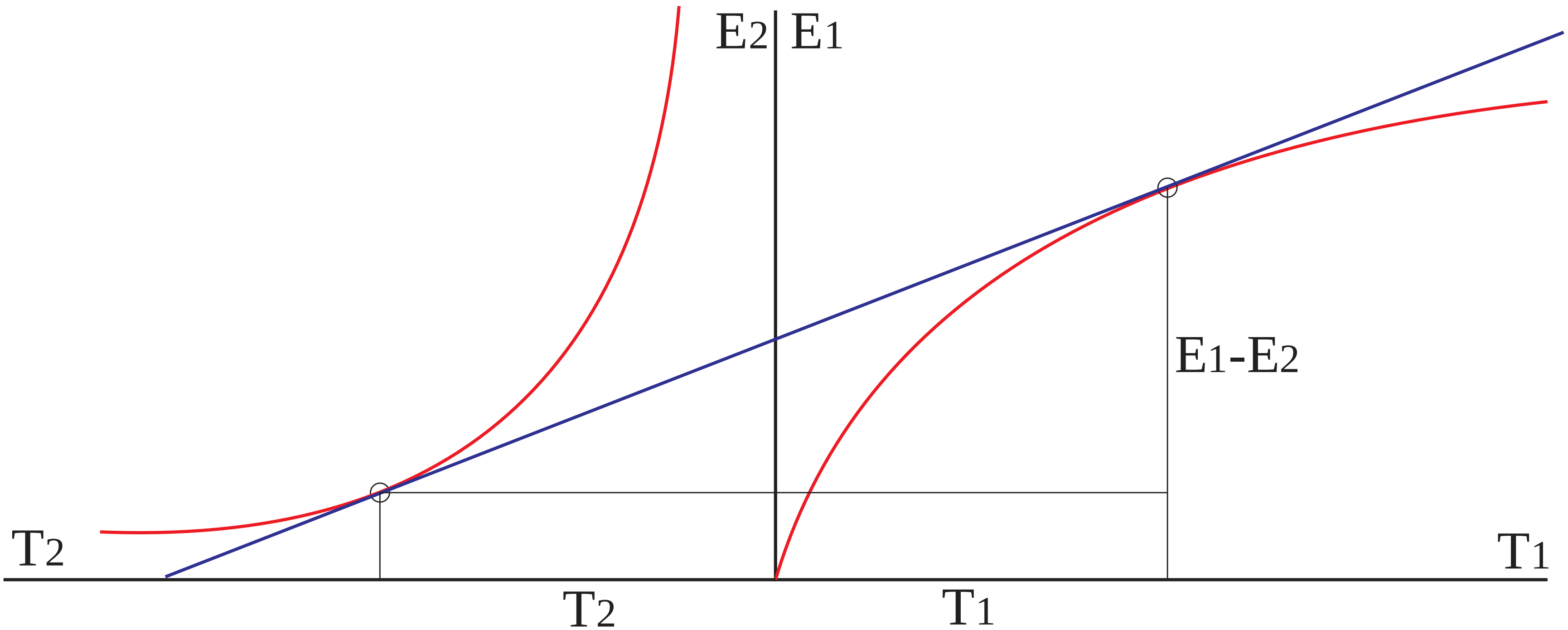


We have already solved the foraging problem (T1).
For a fixed searching strategy, the secant must be tangent to the foraging curve.

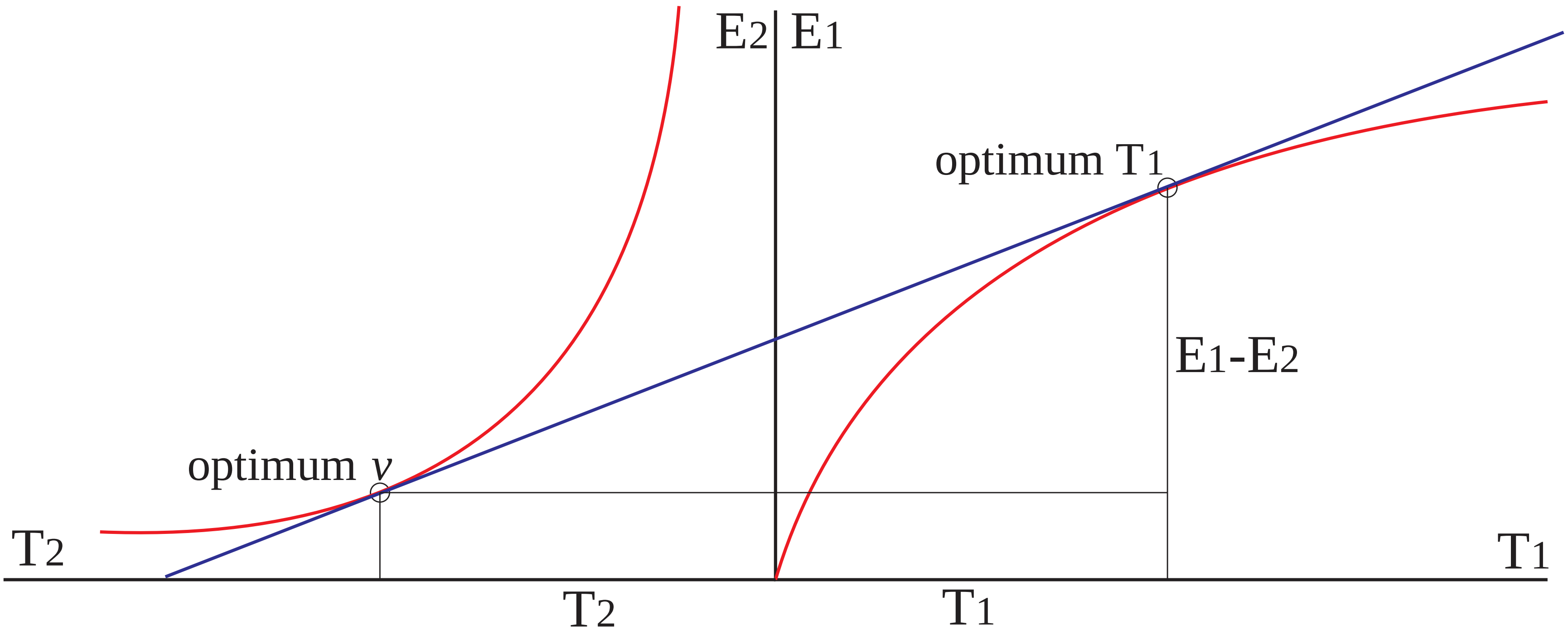


We have already solved the foraging problem (T1).
For a fixed searching strategy, the secant must be tangent to the foraging curve.

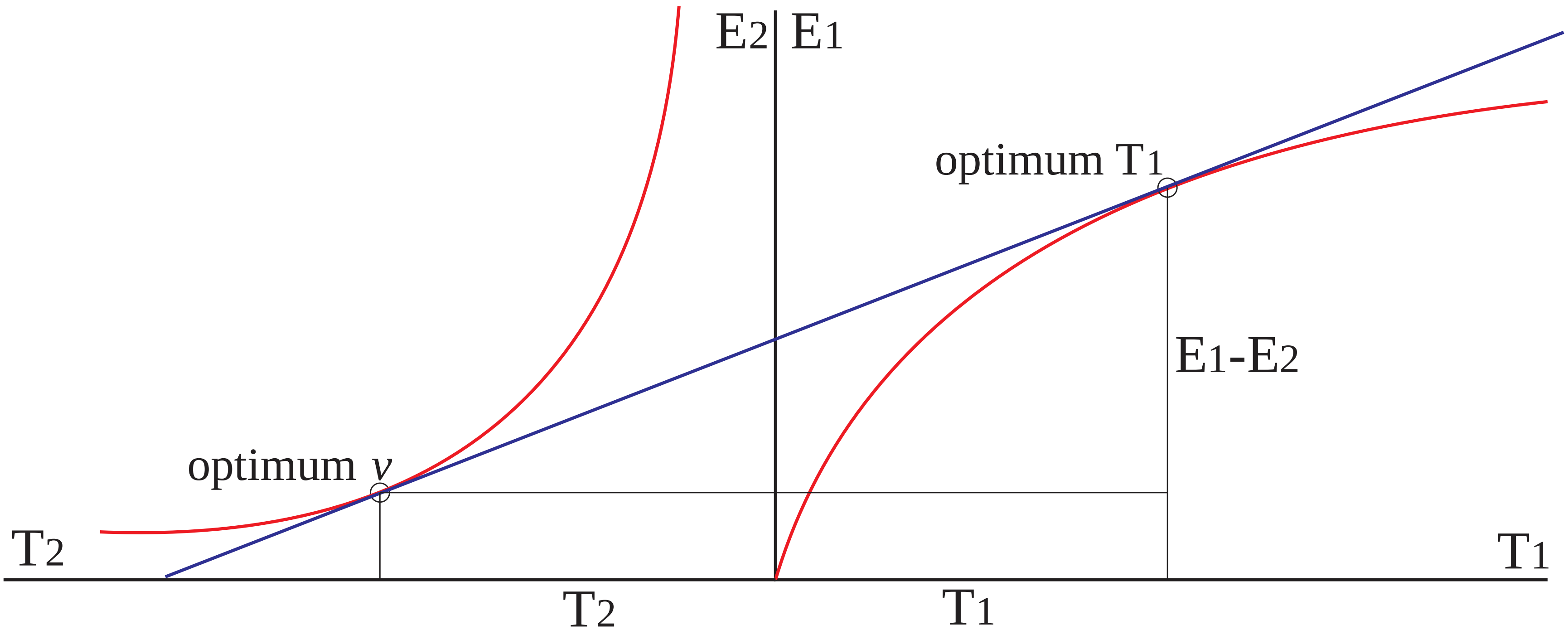
But how do we find the optimal searching time (T2)?



Dare we hope that the optimum line is tangent to both curves?



But of course—what else could possibly be true?



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STRUCTURE

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