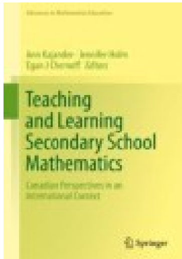

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Powerful Stories: The Hitchhiker's Guide to the Secondary Mathematics Curriculum Landscape

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Abstract Our goals are first to capture a few significant historical moments in the changing face of secondary school mathematics, and secondly to use these to help us decide how we need to move into the future. We decided to speak in a voice that was as personal as possible and that also supported our current research interests, and that of course has conditioned our historical record. Along with that we decided that, rather than write a single piece, we would each write our own thoughts. In particular, Peter would write the central paper, and then Divya, Kariane and Stefanie would each write a reflective response in the currere style (Pinar, W. F., The method of "currere". Paper presented at the Annual Meeting of the American Educational Research Association. Washington, DC, 1975). And of course we would trade ideas at every stage.

Our focus is not on teacher education, nor is it on assessment, though these are both significant components of the shifting landscape, and definitely need continued attention, but we focus here on curriculum. Our long-term objective is to see a high school mathematics curriculum that is driven by what we call powerful stories; as such it would be richer and more sophisticated than what we have at the present and it would be a better platform for the development of "mathematical thinking."

Keywords Life · Narrative · Sophistication · Currere · Mathematical thinking · Dewey · Whitehead

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Peter

The Beginning

On April 9th 1969 at 4 pm I was standing along with a few others outside the high wall surrounding Harvard Yard throwing packets of food, bread and cheese, and juice boxes to some two to three hundred of our fellow students inside. Why was I outside and not inside? I was not sure but was certainly conscious of my inner turmoil. In just over 2 months I was scheduled to be a new father and a new PhD; it was not clear which would come first but both were calling me to be a responsible adult on one side and a revolutionary on the other and anyway why was there a difference between the two.

I lived on Everett Street, facing the wall a couple of blocks away. I was awakened at 5 am the next morning by the shouting and screaming as the Cambridge police took the students away sending 40 of them to Emergency. I went to the window but could not see anything. I looked back at Judith, 7 months along, who thankfully did not wake, and I vowed that in the many professional years ahead of me I would find another way to help steer the change that was so clearly on its way.

> And now the times are changin'.
> Look at everything that's come and gone.
> Sometimes when I play that old six-string.
> I think about you, wonder what went wrong.
> (Adams and Vallance 1984, track No. 6)

The short-lived occupation of Harvard Yard was specifically a reaction against the Harvard administration for its support of the military in the wake of the terrible war in Vietnam, but more generally it was a signal to us all that the change that was on its way would be profound and we simply did not trust the establishment to manage it properly. You see the 60s was an extraordinary decade for education, opening in the wake of the 1957 Soviet launch of Sputnik and closing with Neil Armstrong's 1969 landing on the moon. These events prompted large government investments in science, engineering and mathematics at all levels of education. And one might have asked just how was all that money to be spent?

This question was highlighted that very year with the appearance of *Teaching as a subversive activity* (Postman and Weingartner 1969) and ever since that time, Neil Postman has been one of my gurus. This early somewhat informal work opens with a catalogue of some of the leading thinkers of the day as well as some of the ways education could go wrong:

> The institution we call 'school' is what it is because we made it that way. If it is irrelevant, as Marshall McLuhan says; if it shields children from reality, as Norbert Wiener says; if it educates for obsolescence, as John Gardner says; if it does not develop intelligence, as Jerome Bruner says; if it is based on fear, as John Holt says; if it avoids the promotion of significant learning, as Carl Rogers says; if it induces alienation, as Paul Goodman says; if it punishes creativity and independence, as Edger Friedenberg says; if, in short, it is not doing what needs to be done, it can be changed; it must be changed. (p. 5)

But what exactly is this change to look like? That question has been around for a long time. The goal of this chapter is to highlight a few of the twentieth century milestones for that question and to review briefly some of the reasons that educational change is so difficult and how the situation differs at the elementary and secondary levels. I will also point to some encouraging recent progress.

I end this section with a quote from a 1995 Postman book *The end of education*. [That's a mischievous use of the word "end." One thinks right away of an ending, but in fact Postman intends (mostly) the other meaning of the word: end as goal or objective.]

> What this means is that at its best, schooling can be about how to make a life, which is quite different from how to make a living. (p. x)

I seize here on the precious word "life" and that will form the core concept for the chapter.

The First Half-Century

Alfred North Whitehead (1861–1947), philosopher and scientist, and John Dewey (1859–1952), philosopher and humanist, both wrote definitive essays on education and their ideas are needed today more than ever. For both of them, what happens in the classroom must be significant for the life of the student at that very moment. The fact of the matter is that from their time almost 100 years ago, to the present, this significance has too often been postponed to the future. Here's Whitehead (1929) commenting on the assertion that you cannot do mathematics until you have mastered the technical pieces:

> The mind is an instrument; you first sharpen it, and then use it… Now there is just enough truth in this answer to have made it live through the ages. But for all its half-truth, it embodies a radical error which bids fair to stifle the genius of the modern world… The mind is never passive; it is a perpetual activity, delicate, receptive, responsive to stimulus. You cannot postpone its life until you have sharpened it… There is only one subject-matter for education, and that is Life in all its manifestations. (p. 6)

It is ironic that this dominant idea, that students must wait till university before being confronted with real mathematics, is what is responsible for the fact that so few of them (at most 25%) have anything close to technical mastery of the discipline. This is not only an irony; it is a catastrophe as it engendered the "math wars" that for the past 25 years have pretty much sabotaged any liberal-minded attempt at school curriculum renewal.

It is certainly true that mastery of any complex procedure, whether it belongs in sports, the creative arts, or academics, requires what is often called "automaticity" and this typically requires hours of routine practice. But it is equally true that children (and adults!) love to investigate and discover things and these activities, if they clearly point towards ends that are rich and compelling, can initiate and sustain that

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technical practice. And by the way, humans happen to enjoy time spent in the single-minded company of routine tasks (knitting, musical scales, solving equations) that they can master, especially those that can hide unexpected variations.

For us, Whitehead's point is that the riches of mathematics need to be brought into the life of the student at that very moment. And the big question, the question that inspires this essay, is how is this to be done?

John Dewey also accepts the fact that education must prepare the student for the future and that it is not a simple matter to find the right way to do that, but he also emphasizes that one thing we must not lose is "the organic connection between education and personal experience" (Dewey 1938, p. 8).

> What, then, is the true meaning of preparation in the educational scheme? In the first place, it means that a person, young or old, gets out of his present experience all that there is in it for him at the time in which he has it. When preparation is made the controlling end, then the potentialities of the present are sacrificed to a suppositious future. When this happens, the actual preparation for the future is missed or distorted. ... We always live at the time we live and not at some other time, and only by extracting at each present time the full meaning of each present experience are we prepared for doing the same thing in the future. This is the only preparation which in the long run amounts to anything. (Dewey 1938, p. 20)

Again we ask how we do this and still prepare our students for a world that is technologically hugely more complex than the world of Whitehead and Dewey. Dewey (1938) talks about the importance of framing an intelligent theory (or perhaps a philosophy) of life experience for guiding the growth process, otherwise we are "at the mercy of every intellectual breeze that happens to blow" (p. 21). The problem remaining with us today is to translate Dewey's guiding vision into a concrete curriculum narrative.

In his chapter *The rhythm of education*, Whitehead (1929) describes at length a structure for this narrative. He lays down the three stages of education: Romance, Precision and Generalization and requires that we honour these, and further warns that if in our haste we short-change the critical first one, the second will wither and cannot deliver the ultimate fruit of education: the wisdom of the final stage.

The Past Half-Century

I now jump into the second half of the century where this same life-affirming message is found in the 1976 report on the mathematical sciences in Canada commissioned by the Science Council of Canada (Beltzner et al. 1976). The report noted an "informed opinion that the teaching of mathematics at the elementary and secondary school in Canada is unsatisfactory" and it places a good part of the responsibility on the shoulders of the mathematical community (p. 113). It proposes that the primary aim of the school mathematics curriculum should be "an understanding of what mathematics is" (p. 117) and it cites David Wheeler that "it is more useful to know how to mathematize than to know a lot of mathematics" (p. 119).

A significant outcome of this report was the inaugural meeting in 1977 of the Canadian Mathematics Education Study Group (CMESG/GCDEM), a community of mathematicians, mathematics educators, graduate students and teachers that has met annually ever since and unfailingly provides the collaboration and life-force for much of the Canada's contribution to the study of mathematics education. I will cite one such contribution and that is Whiteley and Davis (2003), a manifesto addressed to the Canadian Mathematical Society asserting that the structure of our K-12 mathematics curriculum "is an obstacle to student learning of mathematics. Over-specified and fragmented lists of expectations misrepresent what mathematics is and militate against deep and authentic engagement with the subject." (p. 83). In addition, the document referenced above all the ability to think mathematically. This is a phrase we are encountering increasingly in the literature, but my experience is that there is little attention paid to mathematical thinking in the "delivered" secondary curriculum.

Powerful Stories

My view is that a significant story, or more generally a collection of related stories that together form a significant narrative, can provide the power needed to propel our mathematics students towards a complete engagement, one that includes technical and conceptual fluency and develops mathematical thinking.

I call such stories "powerful." In literature a story is powerful if it opens the way to a significant human experience. So also in mathematics, a story is powerful if it opens the way to a significant experience of doing mathematics. The stories I look for have a sense of completeness and a natural organic connection to the framework of technical skills that supports them. They must also be accessible but nevertheless have the potential to soar. Gadanidis et al. (2016) call this a "low floor with a high ceiling" (p. 2). Finally, aesthetics plays a huge role in my selection of good stories, indeed it plays the definitive role. Much has been written about "motivation" in the learning of mathematics, and it is certainly true that different students respond positively to different types of experience, but my view is that all students have a natural response to beauty, indeed, aesthetics just might be the universal motivator. Gadanidis et al. (2016) refer to "the aesthetic that makes the experience (of mathematics) human" (p. 2). Certainly mathematicians discover early in life the deep connection between truth and beauty.

Yet when I mention the beauty of mathematics to high school graduates, they often express surprise at the connection. The reason is perhaps that beauty is not displayed as a central feature of school mathematics and thus even when it succeeds in getting into the classroom, it has little staying power. Seymour Papert (1980, as cited in Sinclair 2006) observes that "if mathematics aesthetics gets any attention in the schools, it is as an epiphenomenon, an icing on the mathematical cake, rather than the driving force which makes mathematical thinking function" (p 192). Anne Watson (Sinclair and Watson 2001) comments on this nicely:

> I had a growing disaffection with this pedestrian approach to awe and wonder in mathematics, as if there were common sites for expressing awe, like scenic viewpoints seen from a tourist bus, whose position can be recorded on the curriculum as one passes by, enroute for something else. Spontaneous appreciation of beauty and elegance in mathematics was not, for me, engendered by occasional gasps at nice results, nor by passing appeals to natural or constructed phenomena such as the patterns in sunflowers or the mathematics of tiling. (p. 39)

That's a nice phrase: "scenic viewpoints from a tourist bus." The subtext is: "that's not where I live!" This is the reason that I believe that the investigations we bring into this new curriculum model must be "significant," for example sustained narratives that connect well with the given mathematical curriculum. Otherwise they have little chance of performing as Papert's "driving force." By the way, Papert has some fascinating things to say about mathematics and aesthetics and I recount these in the *Suggestions for further reading*.

Moving Forward—The Challenges

Over the past year we have worked with rich problems of this type in different settings (workshop, classroom) with a variety of students in grades 9–12, mostly in the university preparation stream, and we have learned a lot about what they can do and what they find challenging. Some do not manage to gain a reasonable level of mastery of the methods, but certainly they are all given a significant view of the majesty of a mathematical landscape.

Our ultimate objective is to find 4 years worth of good stories at the secondary level. Certainly these stories must fit the prescribed curriculum, but in moving to a narrative focus, we find ourselves encountering a wider set of mathematical constructions, perhaps intrinsic to the story itself, perhaps raised in the inquiry process by the students themselves, and we need to be open to the pursuit of these. I am thinking here of concepts that are found in probability, geometry, logic, discrete optimization, combinatorics, game theory, stability of physical systems etc. To the extent that such topics find themselves naturally arising, they will of course need some adaptation and teacher buy-in. Most mathematics teachers at the secondary level have a reasonable mathematical background, but they tell me that it takes just about the whole term to cover the mandated technical material, so they would find it difficult to incorporate the activities I am discussing here. My own colleagues in Canadian universities are also wary of these ideas. They already find their students "unprepared" and they suggest that my model could make the situation worse.

I must say that when I look at the list of topics in, say, the Ontario Grade 10 Academic course,[1] I find it hard to believe it takes the whole 4+ months to get that

[1] See McDougall and Ferguson (Part II this volume, para. 1) for a discussion of two of the possible Ontario pathways (Academic and Applied).

done—because I have seen with my own eyes the remarkable things that grade 10 students can do in a week. I can only conclude that in the standard classroom, these students are working at half or quarter throttle. Perhaps the reason for this is that ever since their early years most (but not all) of these students have seen no compelling reason to invest serious effort into the mathematical activities they have encountered.

> There is little space in the conventional normative and normalizing classroom for wonder, for sustained engagement, for obsession, for playful bodies. But we do not seem willing to teach, or even to talk about, the very qualities that animate most mathematicians' life-work. In fact, it is not hard to make the case that precisely the opposite is being taught of mathematics: certainty instead of wonder, detachment instead of engagement, touring instead of dwelling, observing instead of obsessing, scripted performances instead of playful acts. (Davis 2001, p. 23)

There seems to be good evidence that, at least at the elementary level, kids are capable of more mathematical sophistication than the standard curriculum assumes:

> Elementary school teachers work hard to cover grade-specific math curriculum expectations, but what if this is not enough? Ginsburg (2002) suggests that "children possess greater competence and interest in mathematics than we ordinarily recognize" and that they should be challenged to understand big mathematical ideas and to "achieve the fulfilment and enjoyment of their intellectual interest" (p. 7). This position is supported by Joan Moss and her colleagues in their work with functions in Grade 4 (Moss et al. 2008). By developing a stimulating, mathematically rich context for the content that students have to learn, teachers can address grade-specific curriculum expectations while offering students the pleasure of mathematical surprise. Young students, these researchers have shown, benefit from opportunities for using imagination and sensing mathematical beauty. (Gadanidis 2012, p.1)

My view is that in secondary mathematics we are effectively telling our students that we do not think they are clever or imaginative enough to handle the ideas that we, as mathematicians, find interesting and challenging. That strikes me as a smug, elitist attitude that shortchanges the student and the subject itself. One thing I know is that we do not need to fear that our young students will let us down. They are imaginative and resourceful (and even hungry) and simply need to be given problems they can respond to and succeed at. Nor need we fear that our subject will let us down. Mathematics is a veritable goldmine of breath-taking beauty—we just need the courage to bring that into the classroom and (by the way) to stop worrying about whether we are preparing our kids for calculus.

Let me end with a story. Twenty-five years ago Mattel™ marketed a Barbie doll that said "Math class is tough." There was storm of protest and the red-faced company hastily withdrew the doll. That was totally the wrong response; mathematicians should have stood up and pointed out that tough jobs require tough tools and we are lucky that mathematics provides these. These days, trucks and heavy-duty cleansers are praised for their toughness but that is nothing compared with the task of landing a spaceship on an asteroid or designing a code that is easy to implement and hard to break. The fact of the matter is that kids really love the feeling that they are doing a tough job—all they need is some real indication that their hard work is leading to success.

Kariane

This essay for me is a giant puzzle holding many pieces whose assembly might well significantly improve the high school mathematics experience. Towards the end of our *Additional suggestions for further reading* we say: "What we need to do is let go of the imperative that all our kids need mastery of a substantial list of basic technical skills before they can climb the mathematical tree." I believe that this list is so mind numbing that it becomes difficult for anyone, let alone a high school student, to take a step back and see the greater picture. Instead of knowing how all the concepts work together, and how much beauty and power they can generate, many students end up trying to memorize each of them with the hope that it will all work out on the exam.

Reflecting on my own experience as a high school student, that certainly was the case for me. My 16-year-old self was anxious, unaware of her strengths, but certainly aware of her weaknesses—and one of these was mathematics. I was an average student until I failed miserably the first exam of Secondary 5. When I managed to get over the shock and the shame, I realized two things. The first is that if mathematics was required, it must be doable. The second is that I cannot be that dumb. Hence, success was within my reach, and through introspection, I realized that memorizing mathematics was probably not working for me. That is what I have been doing instinctively for years, but I had no fundamental understanding of any of it. Therefore, *I learned how to learn math*, and underlying that important process was a change in my motivation: I was now learning mathematics for its own sake. Middleton and Spanial (1999) would call this *mastery goals*. On the other hand, *ego goals* define success in a discipline relative to others. In terms of achievement, "students with mastery goals tend to perform better than those with ego goals regardless of the learning situation" (Middleton and Spanial 1999, p.74). This could explain my shift from underachievement to being successful in mathematics.

The question that remains is how we might replicate this for other students. Studies showed two important things regarding this question. The first is that students' intrinsic motivation (linked to mastery goals) towards mathematics decreases significantly throughout high school (Gottfried et al. 2007, p. 325). The second important lesson is that "the decline in academic intrinsic motivation is not a general developmental or ontogenetic one, nor is it inevitable" (Gottfried et al. 2001, p.10).

One way to remedy this decline is by implementing an inquiry-based classroom. By using such a model, students "are less likely to develop ego goals than are students in more traditional classrooms" (Middleton and Spanial 1999, p. 74). Moreover, they tend to believe that success in mathematics is defined by their attempts to understand and explain their thinking (Middleton and Spanial 1999, p.74). My hope is for discovery and investigation to be at the centre of the new face of mathematics in Canada.

Divya

While thinking about what to write for this chapter, I stumbled upon the article titled, "School mathematics as a special kind of mathematics" (Watson 2008). One of several responses to Watson's thoughts was Mendrick (2008) who stated three reasons for why she believed there will always be a difference between the two types of math. These were:

- School students do not get paid for doing mathematics
- They do not apply for opportunities to do it
- Even when they have an identity that is invested in being good at it, mathematics never defines them in the way that one's employment does

Mendrick's list made me think back to Peter's comments. He stated:

> My view is that in secondary mathematics we are effectively telling our students that we do not think they are clever or imaginative enough to handle the ideas that we, as mathematicians, find interesting and challenging. That strikes me as smug, elitist attitude that short-changes the student and the subject itself.

Like Peter, I also find that we are creating this "elitist attitude" with mathematics. Whether or not we feel we can accept Mendrick's three differences, I feel that they also show this attitude; that mathematicians are simply too different from their students. I find this to be a huge problem, as it can create many misconceptions for students, specifically regarding creativity and discovery in mathematics.

While I am now following a career in mathematics, in high school I fell victim to these misconceptions. Throughout my life, I have always been interested in mathematics. This was encouraged by my family, who promoted my curiosity in the subject. However, school did not. As I went through my high school years, I felt less interested in mathematics as I was constantly feeling as though I was not challenged. Therefore, like the students that Peter mentioned, I found physics more interesting, and so I decided to major in physics in university. However, my constant curiosity made me keep up with mathematics, and I eventually transferred into Applied Mathematics.[2] If it was not for the opportunities I was given and the encouragement from my family, I would never have found my place in mathematics, and seen how much there is to discover.

Finally, I want to come back to Mendrick's thoughts. Even given that school mathematics will not be exactly like the mathematics done by mathematicians, there should not be such a disconnect between the two. Just like mathematicians, students

[2] This term refers to one of the two course pathways in Ontario secondary classrooms (Applied and Academic).

should feel encouraged to use their curiosity and creativity to help them learn challenging mathematics. Perhaps then can we put an end to this elitist attitude.

Stefanie

Our research aims to enrich the learning of mathematics at the secondary level by engaging students with powerful stories. Our recent project presented grade 10 students with rich problems involving transformations and matrix multiplication (see Fig. 1). My encounter with these students clearly pointed to a lack of drive and interest towards their usual mathematical activities. The students would ask, "What is the purpose of learning this stuff? I'll never use it again outside of this classroom!" Other students failed to connect our activities with their idea of mathematics as they would ask, "At what point will we begin to do math?" I had similar questions in my own past, as had the authors of the previous two testimonies. It is not easy to answer these questions but they certainly suggest that the learning of school mathematics could be improved.

We all strive to find a purpose in our efforts and feel there is a reason for our time spent learning. We call the activities of our project "powerful stories" because they are designed to have the power to guide the learner towards the feeling of a worthwhile purpose. This seems to be lacking in the material presented in the curriculum today.

Mehta et al. (2016) interviewed four Fields medal recipients to highlight how mathematicians view mathematics from a creative and artful perspective that is lacking in the high school curriculum. The authors suggest that the current curriculum limits students' ability to wonder, to develop a sense of the field of mathematics and to envision themselves as mathematicians.

> These profiles [of the four mathematicians] tell us that success in mathematics comes with passion and play, and from seeking connections across fields and disciplines. They provide a very different view of mathematics—as a living, artistic, organic structure, that mathematicians actively construct in order to find truth and beauty in the world. We believe that this view of mathematics has significant implications for how we think of teaching and learning in this domain. It offers a novel and humanistic way of thinking about how to engage educators and learners in mathematical ideas. (p. 18)

We hope that by providing students with powerful stories, we will be able to help them to encounter mathematics from this perspective, be more engaged by the material, and be motivated to endure longer as they construct their own reasons for pursuing mathematics (Fig. 1).

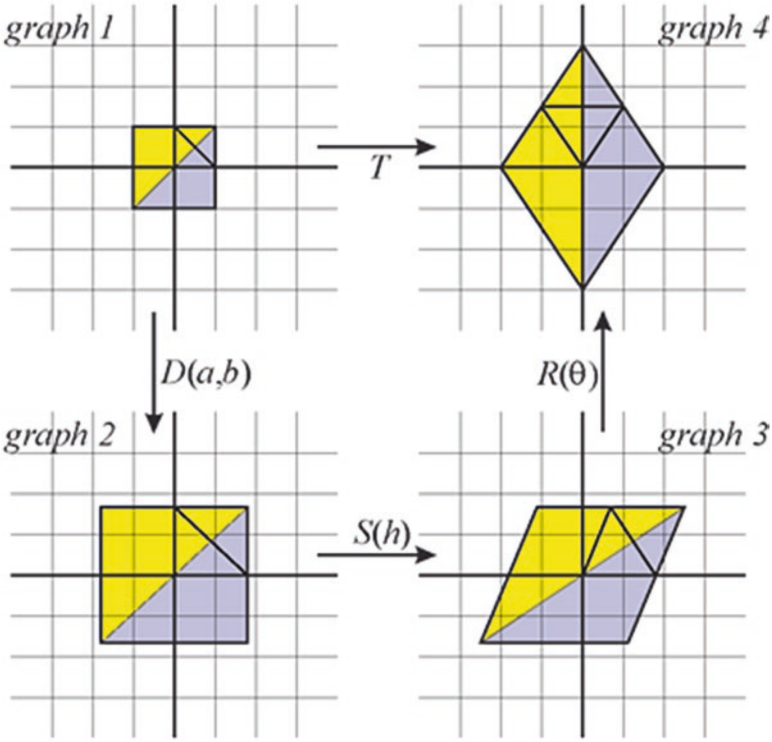

Fig. 1 Factoring transformations (www.Math9-12.ca, Transformations Example 6)
This example embraces both computational thinking (CT) and spatial reasoning (SR) and is designed for Ontario Grade 10 or 11 Academic (i.e., university mathematics/science preparation). Here T is a linear transformation, and students are required to factor it as a given composition of what we have taken as "basic" transformations—dilations D, rotations R and horizontal shears S. The task is to find the parameters a, b, h and θ. In a sense, the basic transformations act as the "primes" of the system and the problem is to "factor" T as a "product" of these primes. There are two approaches, *geometric* using coordinate geometry and triangle trig, and *algebraic* using matrix multiplication and solving equations.
At first sight this seem like a difficult problem, and it does take some focused thinking to put all the pieces together. Having done that with a few examples, one can almost see an "algorithm" emerge; it is however sophisticated, both in its geometric and algebraic form, and it would be difficult to implement without a good grasp of the basic ideas and techniques.
For this particular transformation T, the reader will notice that there is an obvious and simpler factorization as a rotation followed by a dilation. But the sequence given here—a dilation followed by a shear followed by a rotation—is not so easy to find. The answer is $a = 5/2$, $b = 12/5$, $h = 7/24$ and $\tan\theta = 4/3$.

Additional Suggestions for Further Reading

The Role of the Aesthetic in Mathematical Discovery

Much has been written about the beauty of mathematics, that for mathematicians it is a significant experience and the deep pleasure that it brings motivates them in their work. Based on this, it is a great pity that this experience is rarely to be found in the school classroom. But there is a strand of philosophical thought going back at least to Poincare that makes a stronger and more essential argument; it says that the aesthetic is a central component of the mathematical experience in that serious mathematics simply cannot be done without it. As a result, the failure to place the aesthetic experience at the centre of the mathematics classroom is more like a disaster, certainly a token of failure for folks who would construct a more sophisticated curriculum. There is quite a large literature that discusses this idea (Google: Poincare mathematics aesthetics) but I find Seymour Papert's (1980, p. 190–197) presentation of the mechanism at work here to be the clearest and the most intuitive.

Here is the idea. The mind has two components: the conscious which operates logically, and the unconscious which does not. Problems of a mathematical nature, when received, go to the conscious to be worked on and are often simply solved. But problems which are harder or less familiar, or even less well-formulated, after some preliminary analysis and a bit of struggle, are dispatched to the unconscious where they might reside for some time. During this interval, while the conscious mind is occupied with the business of living, these problems are quietly worked on but in quite a different way. The unconscious mind uses aesthetic criteria, the elegance, harmony and order found in patterns, to make value judgements, to decide what to transform and how, what to accept and what to reject. At some point, when it is "ready," it throws the results back to the conscious mind, often taking it by surprise (Poincare tells a now famous story of this happening as he was stepping onto a bus). But armed with this transformed version of the problem, the mind can now apply its prowess with logic to evaluate the configuration and hopefully move the solution forward. If this story provides a reasonable account of reality, it must follow that both aspects of mind deserve to be trained and that a child ought to receive both a logical and an aesthetic education.

Our objective here is to conclude with a mention of the many Canadian researchers who have emphasized the fundamental role played by the aesthetic in the mathematical experience and have worked to carry this idea into our secondary schools. A brief search of the literature suggests that this is a hopeless task; there is so much wonderful work, one cannot decide where to stop. I (PDT) will mention a few major influences on my own life. First mention goes to my former PhD student (shared with my colleague Bill Higginson) Nathalie Sinclair (Simon Fraser) whose wonderful book *Mathematics and beauty* (2006) argues that students are fundamentally aesthetic beings and provides examples and activities to move the curriculum in this direction. Ed Barbeau (University of Toronto) places the aesthetic experience at the centre of a rich array of problems he has developed over the years. He has argued, along with many others, for an investigative "capstone" course in the final year of high school, perhaps with the theme of optimization. Such a course could happily replace the standard functions/calculus grind and at the same time provide excursions into geometry, probability, combinatorics, and game theory—all excellent areas for the development of an aesthetic mathematical experience. Finally, I cite Walter Whiteley's (York University) persistent inspirational work building a case for the return of geometry. These personal mentions are followed by a host of others. Many of these can be captured in their work and from east to west I mention *Math circles* (Nova Scotia), and the amazing magazine publications *Accromath* (Quebec) and *Pi in the sky* (PIMS). I find these sites remarkable for the deep, beautiful and accessible mathematics they provide. There is a cruel paradox here. These publications are rich enough to easily fuel the secondary mathematics curriculum and they would give our students an inspiring and sophisticated experience far removed from the text-books that are currently in use. But our current ideas of the nature and structure of the school mathematics curriculum would need a fundamental change.

Embracing a Sophisticated Experience in the School Mathematics Classroom

Three prominent Canadian websites that embrace the significance of rich mathematical structures in school mathematics are:

Computational Thinking in Math Education www.ctmath.ca/about/
Math for Young Children (M4YC) http://www.mathforyoungchildren.ca/.
Spatial Reasoning Study Group (SRSG). http://www.spatialresearch.org/group

The much-cited essay Lockhart's *A mathematician's lament* (2002) imagined what would have happened if music instead of mathematics had been the subject considered essential for all students to "learn." The idea here is that school mathematics would perhaps be richer and more artistic if we let go of the idea that it was so important for the future of all our students. Well, I would like to say that differently. Mathematics is as important for all our students as English and history and science and music and art and physical education, and the list could go on. What we need to do is let go of the imperative that all our kids need mastery of a substantial list of basic technical skills before they can climb into what Dan Kennedy (1995) calls the mathematical tree. I would go so far as to predict that if we effectively let go of that list, and spend our mathematics time more like they spend time in art class, the overall level of technical proficiency will actually go up rather than down.

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