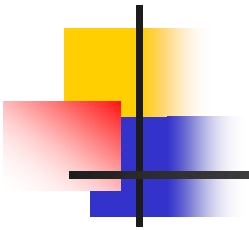

Simon Fraser University 2007

Will the real Q please step forward?

Peter Taylor
Dept Math&Stats
Queen's University

May 23, 2007

Only two things are infinite, the
universe and human stupidity,
and I'm not sure about the former.
-- Albert Einstein (1879-1955)


Picasso
25.7.61.

The way of the artist

Q and the artist can walk together

Q and the artist can walk together

In fact, that’s how it needs to be.

The meeting Courbet 1864

A third type of course eligible for Q designation will be designed especially for students in the Humanities and Fine Arts. The goal of such courses will not be simply to nurture traditional math skills. Such courses will aspire to the greater challenge of deepening the understanding and appreciation of quantitative and formal reasoning, their ubiquitous utility, and their creative potential. We view such courses as focusing on the relation between (a) concepts and structures communicated through numbers and other systems of abstract representation (such as formal languages, programming languages, geometries, graphs) and (b) fostering students' ability to engage more effectively with the subject matter of their respective programs and practical everyday situations. Such courses need not focus primarily on quantitative or formal reasoning methods, but should give significant exercise to such techniques through model building and problem solving, both in class and in course assignments.

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Q and the artist

What will they talk about?

sophistication

The meeting Courbet 1864

Nothing Gold Can Stay
Robert Frost

Nature's first green is gold,
Her hardest hue to hold.
Her early leaf's a flower;
But only so an hour.
Then leaf subsides to leaf.
So Eden sank to grief,
So dawn goes down to day.
Nothing gold can stay.

wholeness

context
meaning

structure

Quantitative curricula traditionally take their structure from the logic of the technical skills.

But if designed by an artist the structure would be framed at a higher level.

thematically

contextually

SONNET 73

That time of year thou mayst in me behold
When yellow leaves, or none, or few, do hang
Upon those boughs which shake against the cold,
Bare ruin'd choirs, where late the sweet birds sang.

In me thou seest the twilight of such day
As after sunset fadeth in the west,
Which by and by black night doth take away,
Death's second self, that seals up all in rest.

In me thou see'st the glowing of such fire
That on the ashes of his youth doth lie,
As the death-bed whereon it must expire
Consumed with that which it was nourish'd by.

This thou perceivest, which makes thy love more strong,
To love that well which thou must leave ere long.

freedom

restraint

Those Winter Sundays
Robert Hayden

Sundays too my father got up early
and put his clothes on in the blueblack cold,
then with cracked hands that ached
from labor in the weekday weather made
banked fires blaze. No one ever thanked him.

I'd wake and hear the cold splintering, breaking.
When the rooms were warm, he'd call,
and slowly I would rise and dress,
fearing the chronic angers of that house,

speaking indifferently to him,
who had driven out the cold
and polished my good shoes as well.
What did I know, what did I know
of love's austere and lonely offices?

sophistication

wholeness

context

meaning

structure

freedom

restraint

wonder

mystery

magic

paradox

chaos

sophistication

wholeness

context

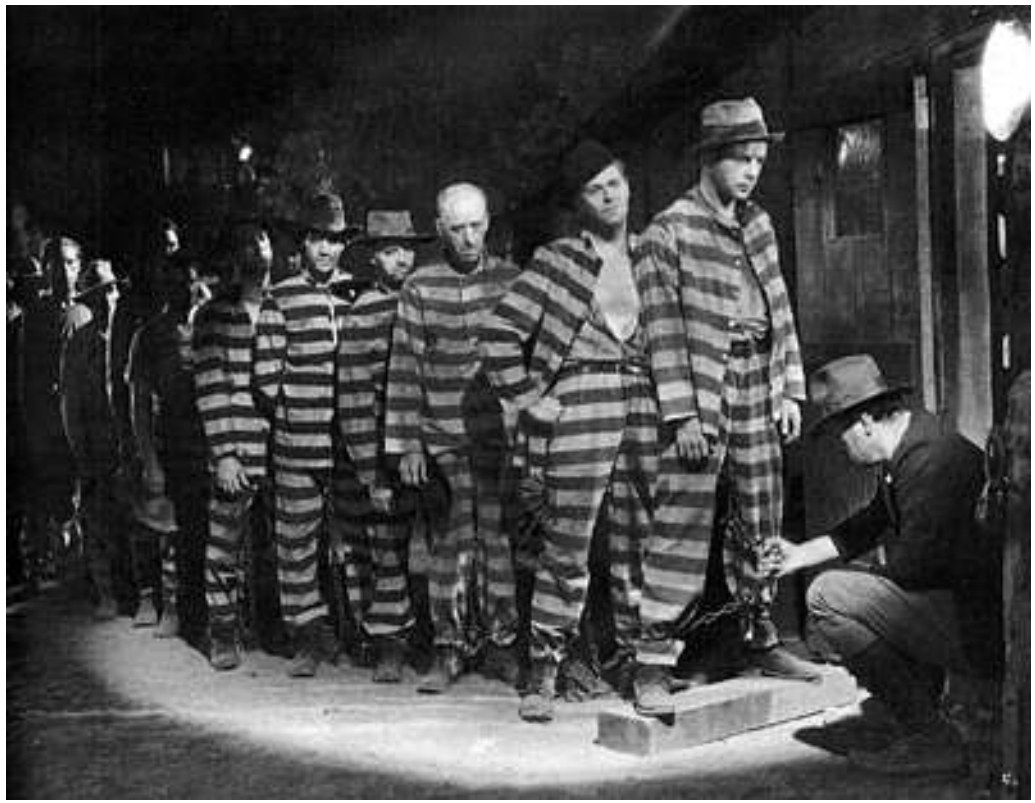
meaning

structure

freedom

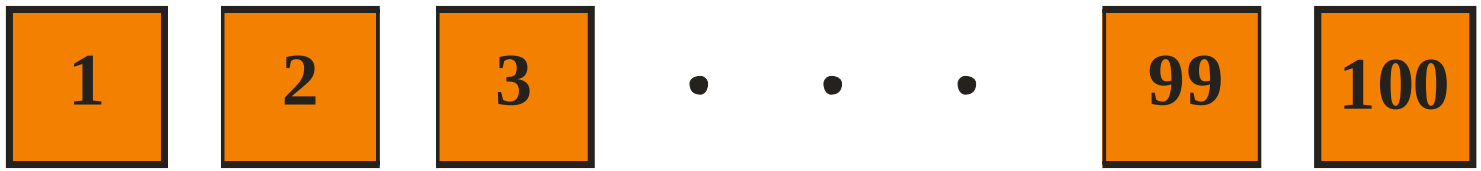
restraint

The prisoners and the boxes



100 prisoners
They will all be executed at dawn
unless they succeed in the following trial.

100 prisoners each with a different name
Prisoners enter the room one at a time
In the room there are 100 boxes
each containing the name of one prisoner

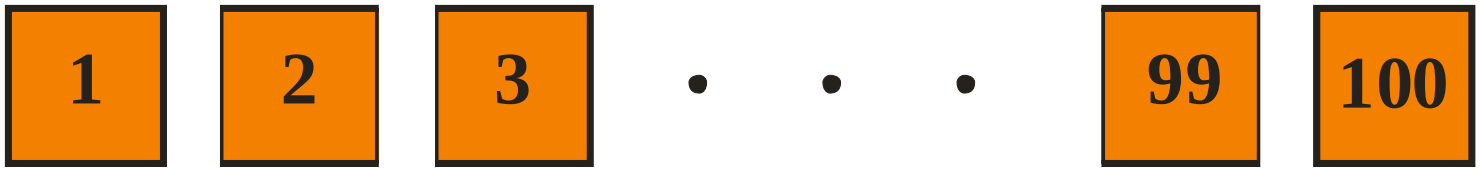


Each prisoner opens 50 boxes of his choice

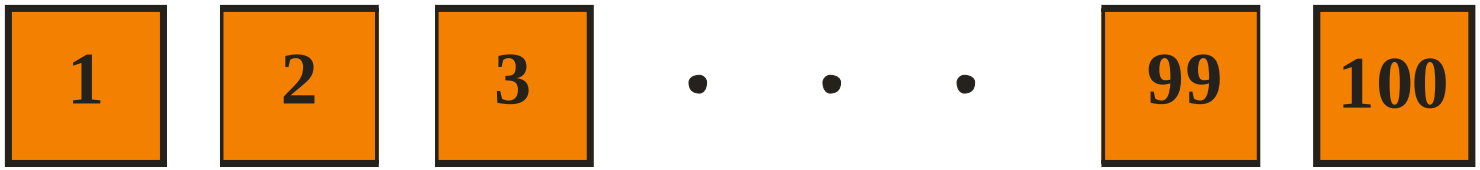
In doing so he may find his own name
Or he may not

He then leaves returning all boxes to their original state

And has no further contact with the other prisoners!

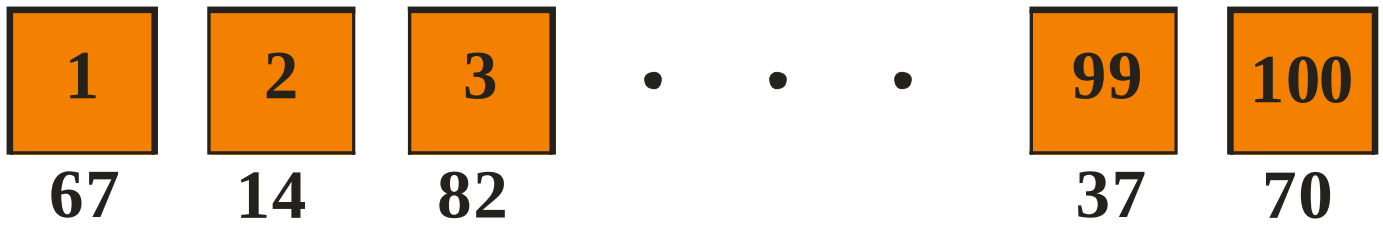


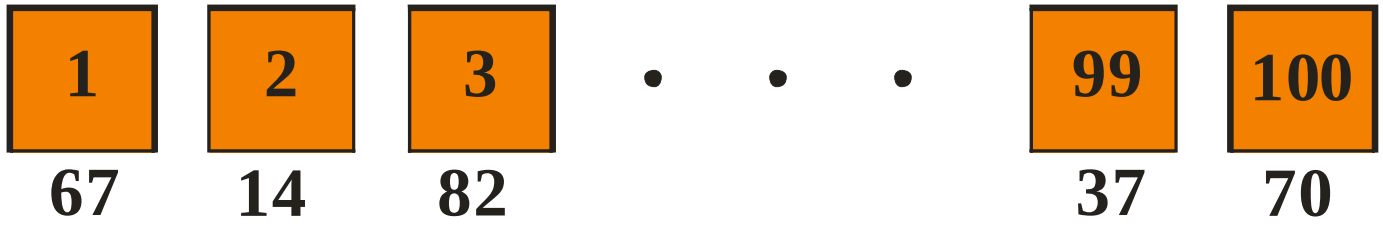
The prisoners will **all** be reprieved
if they **all** find their own name



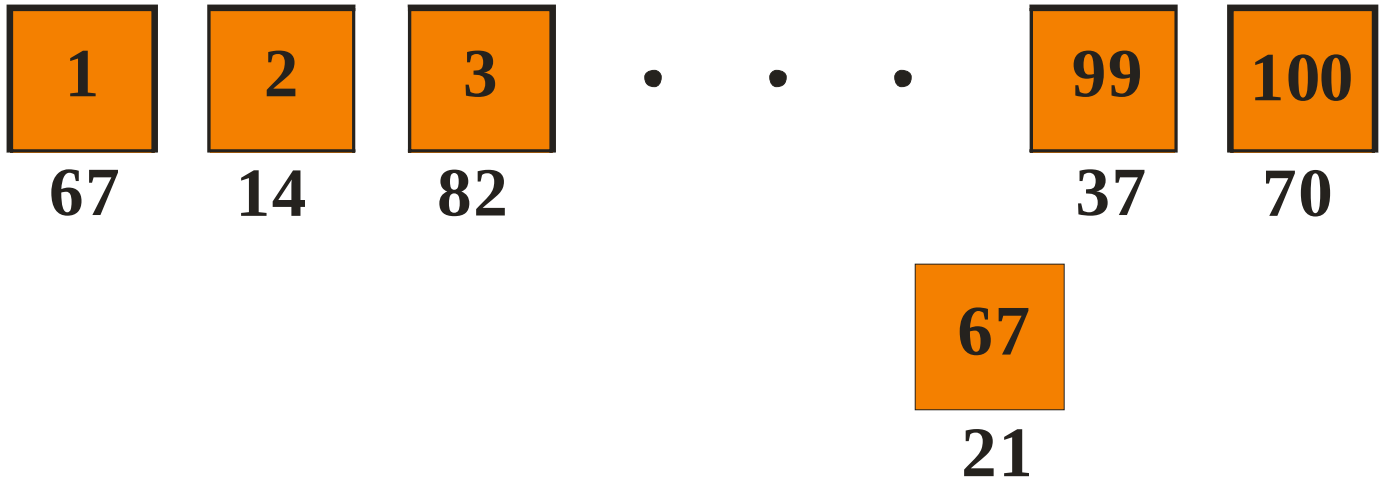
The prisoners will **all** be reprieved
if they **all** find their own name

Otherwise if **at least one** fails to find his name
they all die at dawn

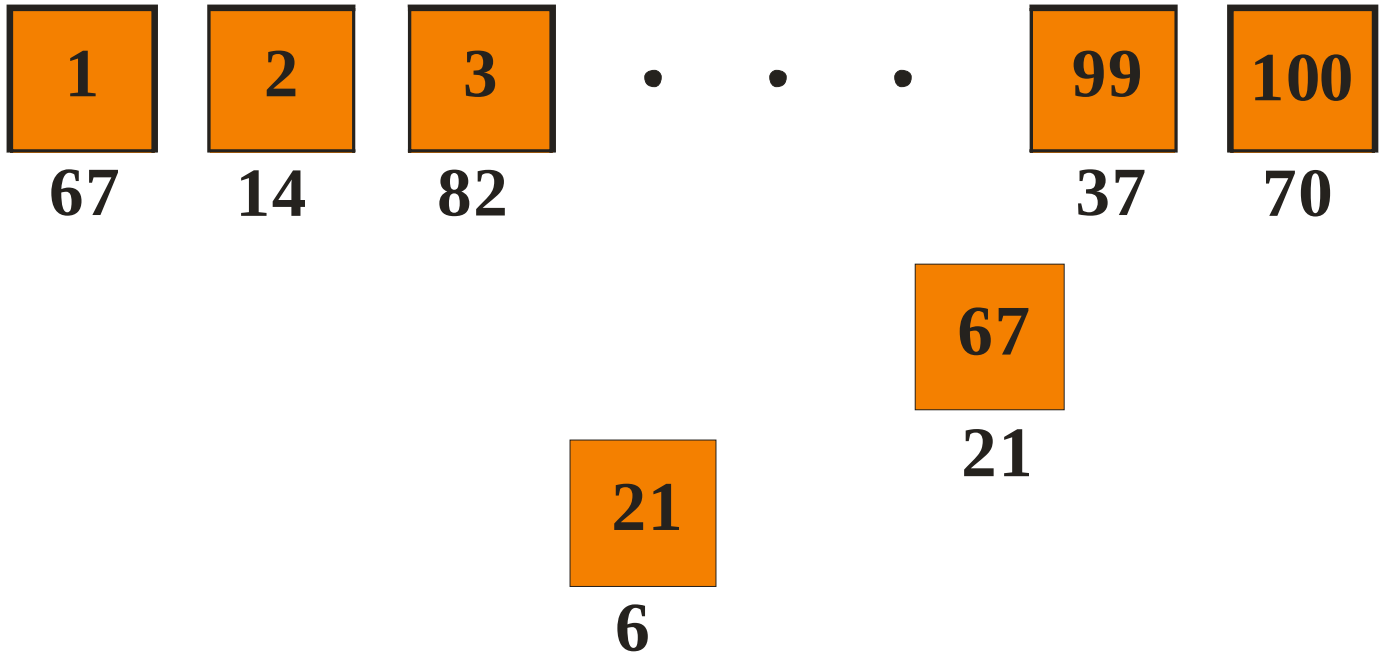

1
2
3
99
100
67
14
82
37
70



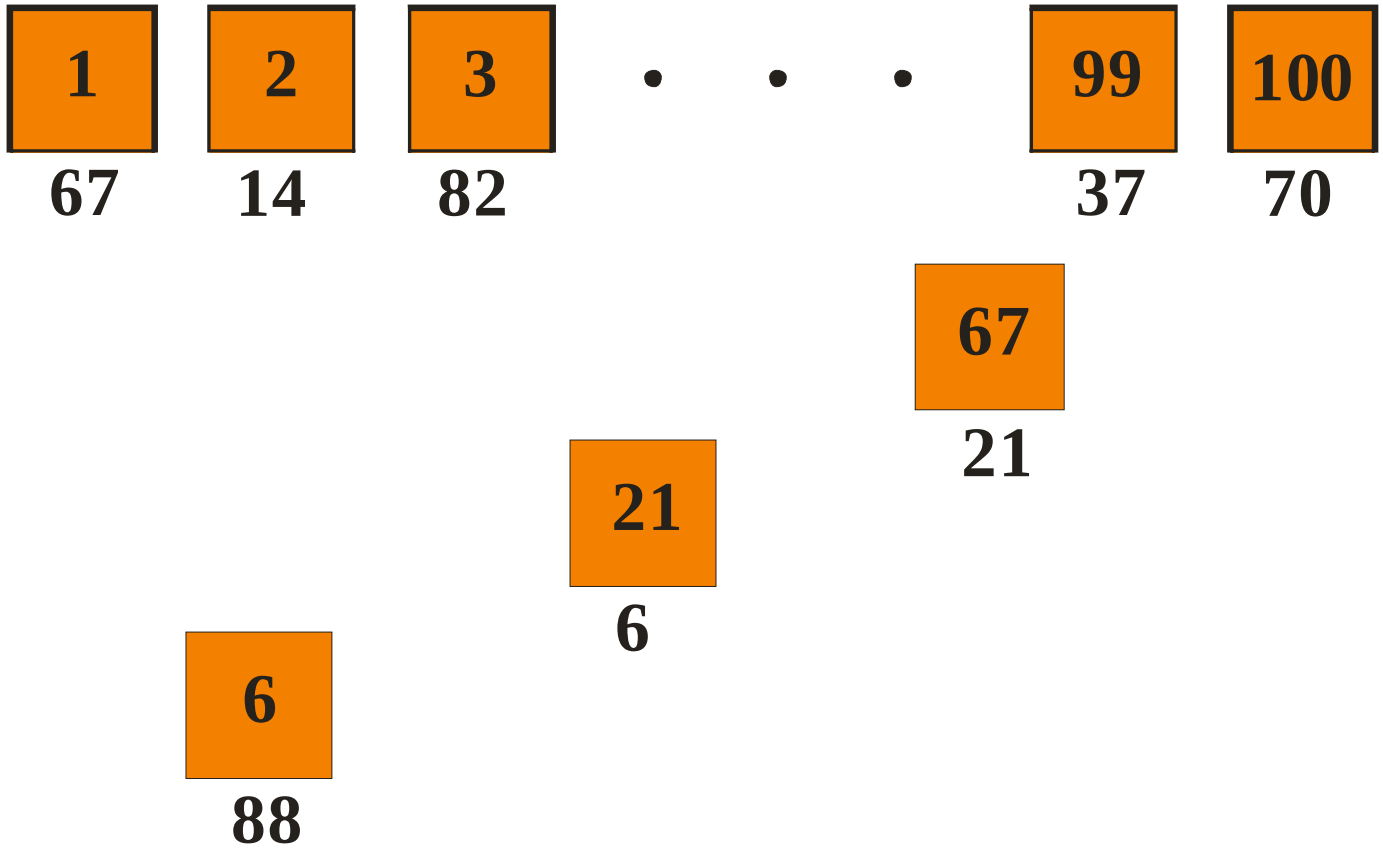
Prisoner 1 opens: 1



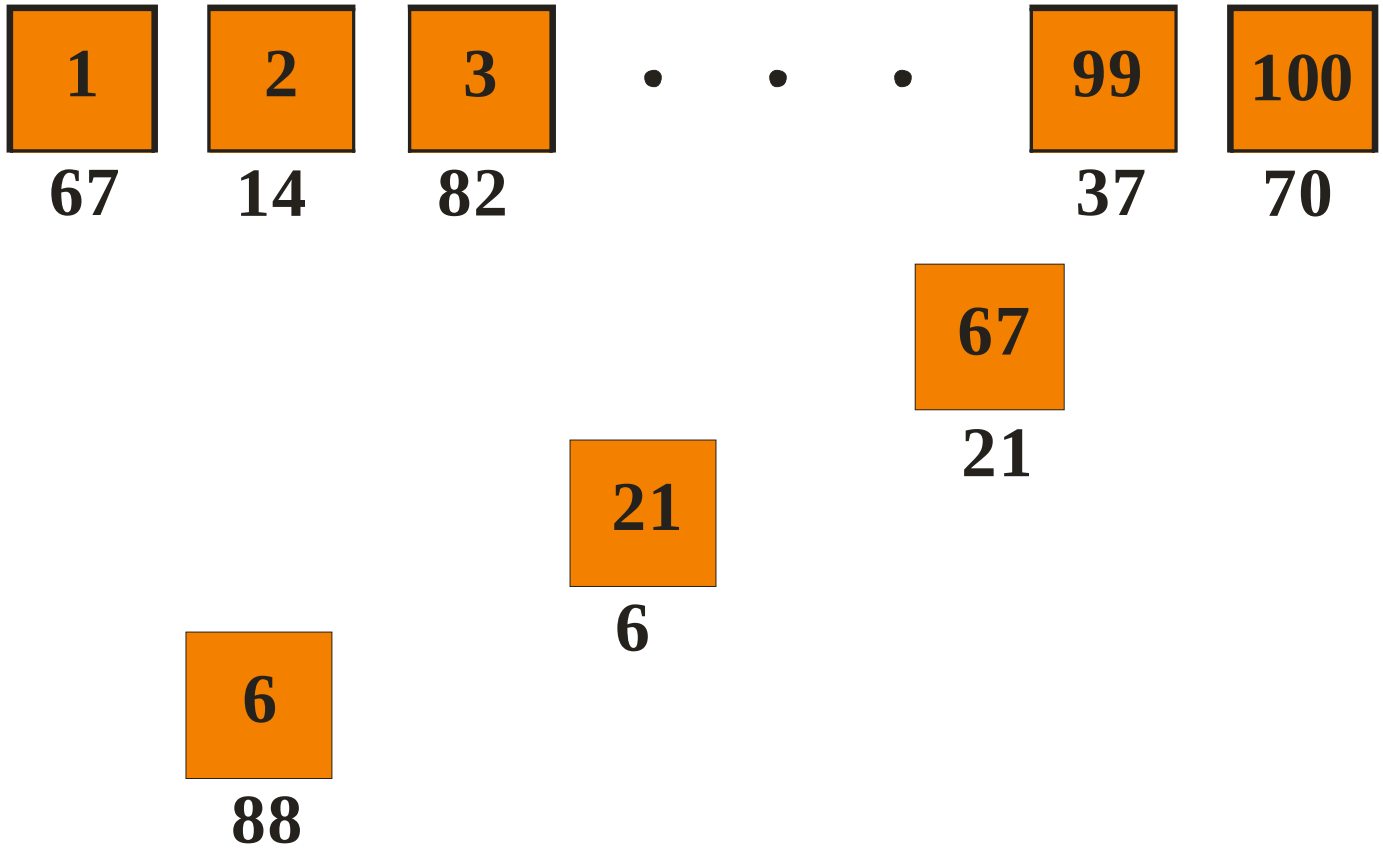
Prisoner 1 opens: 1 67



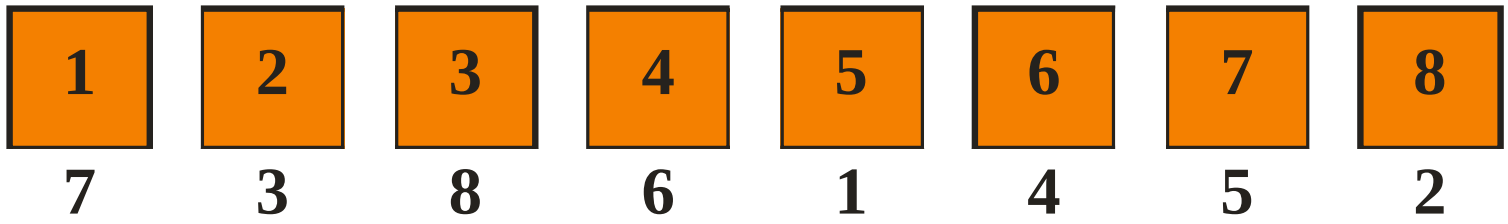
Prisoner 1 opens: 1 67 21



Prisoner 1 opens: 1 67 21 6



Prisoner 1 opens: 1 67 21 6 88 …


1
2
3
4
5
6
7
8
7
3
8
6
1
4
5
2

1	2	3	4	5	6	7	8
7	3	8	6	1	4	5	2

#1: 1 7 5

1	2	3	4	5	6	7	8
7	3	8	6	1	4	5	2

#1: 1 7 5 

#2: 2 3 8 

1	2	3	4	5	6	7	8
7	3	8	6	1	4	5	2

#1: 1 7 5 

#2: 2 3 8 

#3: 3 8 2 

1	2	3	4	5	6	7	8
7	3	8	6	1	4	5	2

#1: 1 7 5 

#2: 2 3 8 

#3: 3 8 2 

#4: 4 6 

1	2	3	4	5	6	7	8
7	3	8	6	1	4	5	2

#1: 1 7 5

#2: 2 3 8

#3: 3 8 2

#4: 4 6

My students playing the 7-hat game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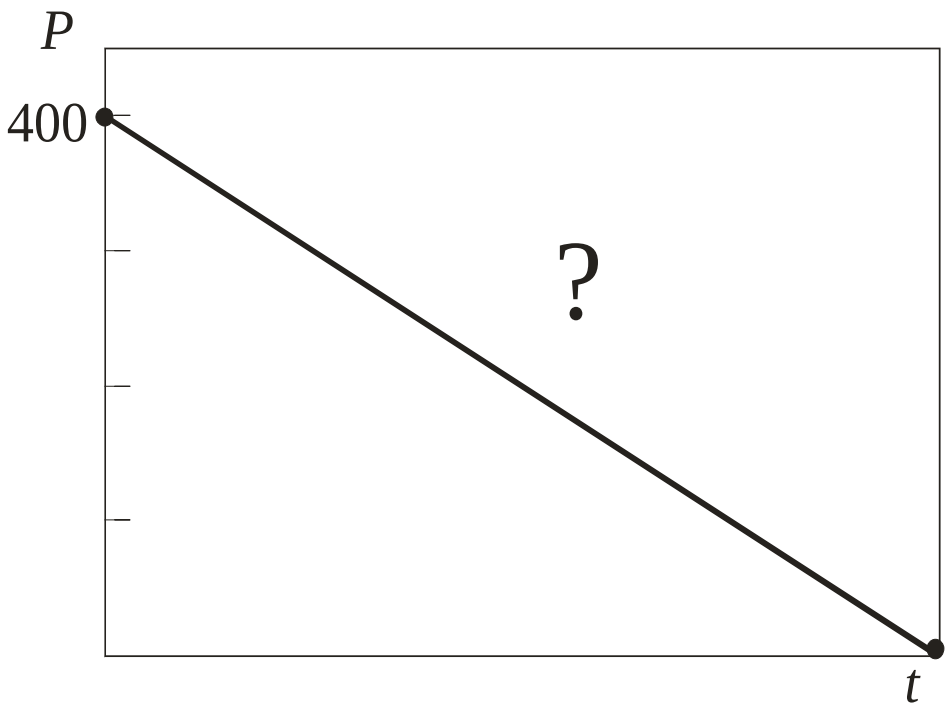
Tire pressure

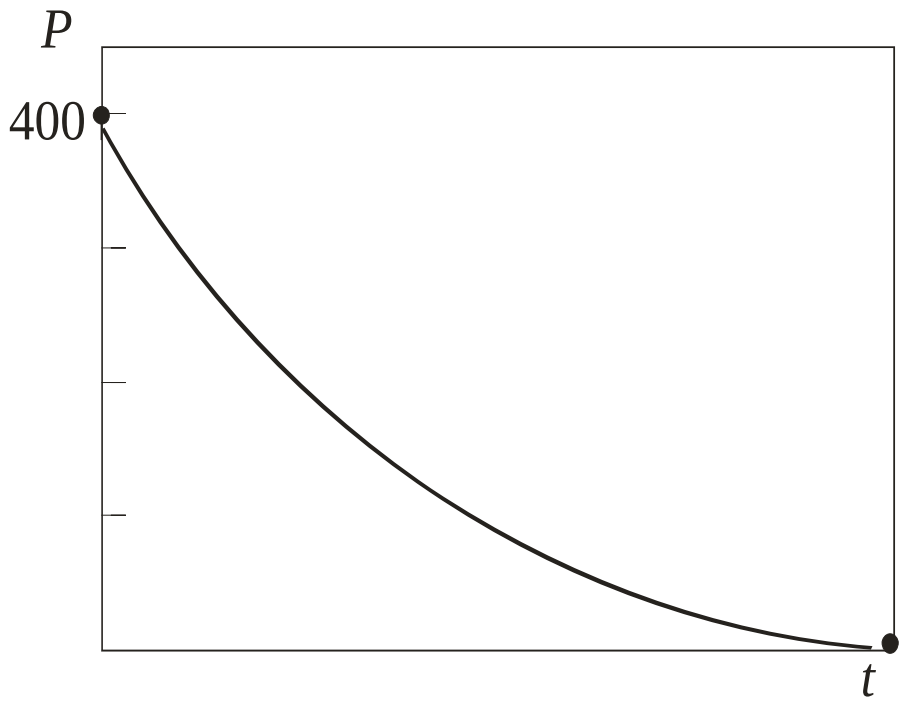
You have a hole in your tire. You pump it up to P=400 kilopascals (kPa) and over the next hour it goes down till the tire is quite flat. Draw what you think the graph of tire pressure P against time t should look like.

What will the graph look like?

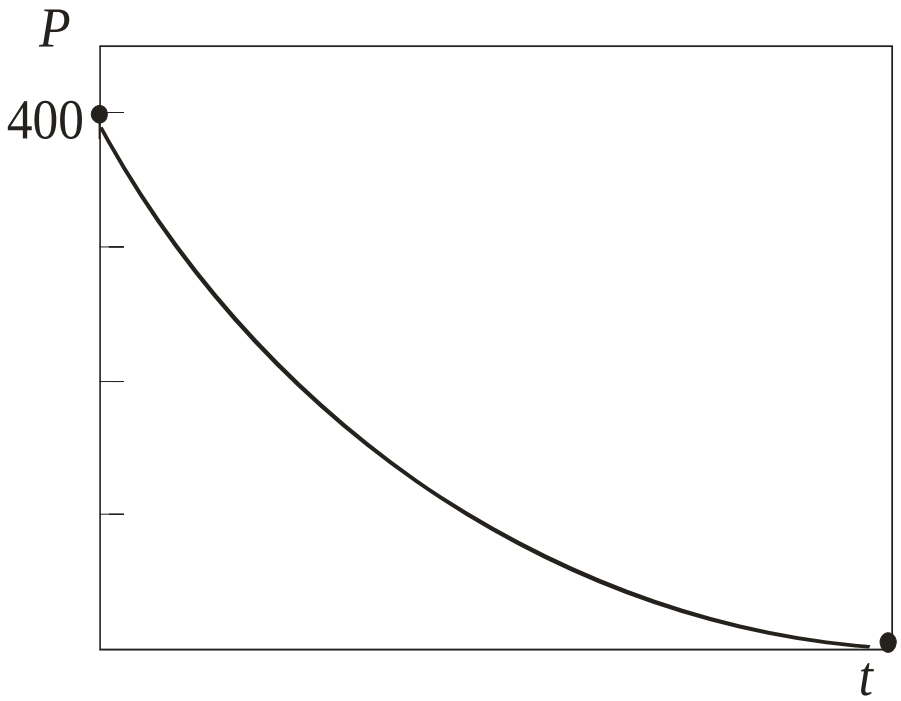
What will the graph look like?



Maybe it looks like this.



But what kind of graph might this be?



Suppose your tire has a small leak.
Suppose the starting pressure is 400 kPa.
1 minute later the pressure is 384 kPa—
a one-minute loss of 16 kPa.

Now leave it for a while until it's 200 kPa.
Half of what it started with.
So here's the question—
How much will it lose in one minute now?

Suppose your tire has a small leak.
Suppose the starting pressure is 400 kPa.
1 minute later the pressure is 384 kPa—
a one-minute loss of 16 kPa.

Now leave it for a while until it's 200 kPa.
Half of what it started with.
So here's the question—
How much will it lose in one minute now?

Answer: Half as much—8 kPa

Here's the big idea:

The one-minute loss is proportional to the amount.

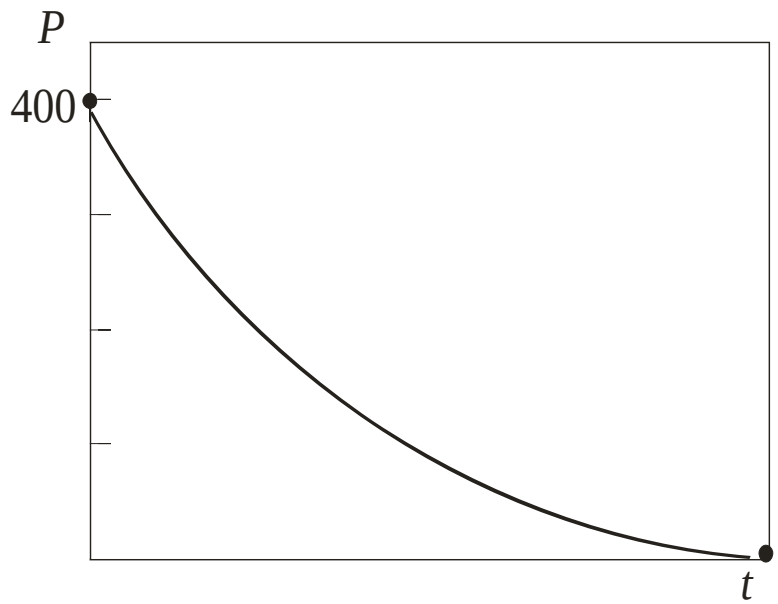
Here's the big idea:

The one-minute loss is proportional to the amount.

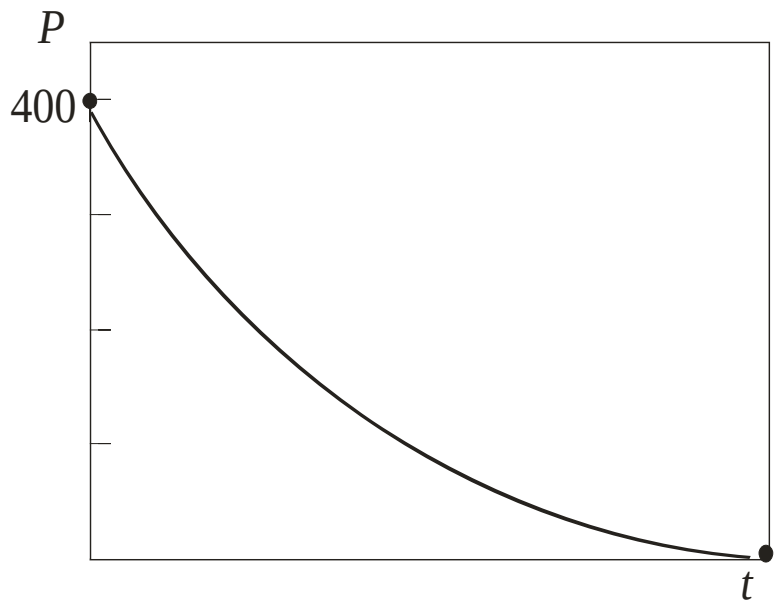
The one minute *percentage* loss is constant.

Starting pressure: $P(0) = 400$ kPa.
Every minute, P decreases by 4%.

Find $P(t)$ the pressure after t minutes.

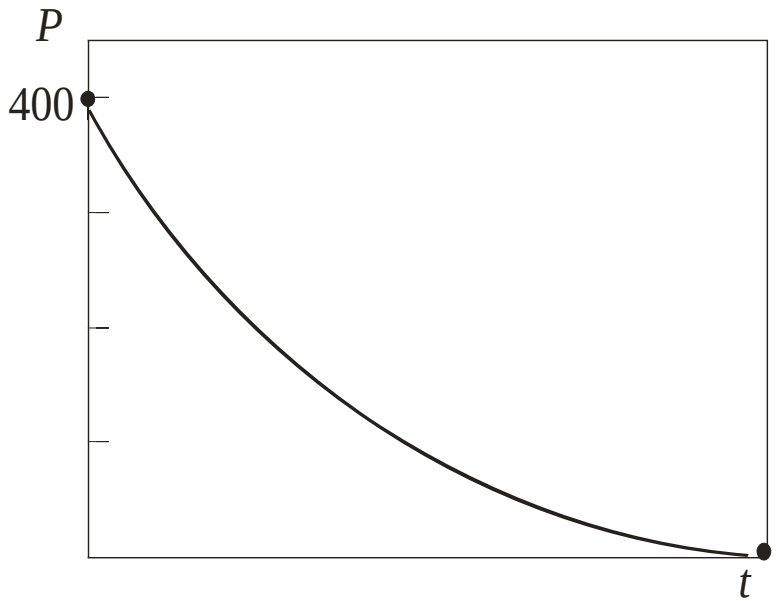


Starting pressure: $P(0) = 400$ kPa.
Every minute, P decreases by 4%.
That means every minute P is multiplied by 0.96.
Find $P(t)$ the pressure after t minutes.

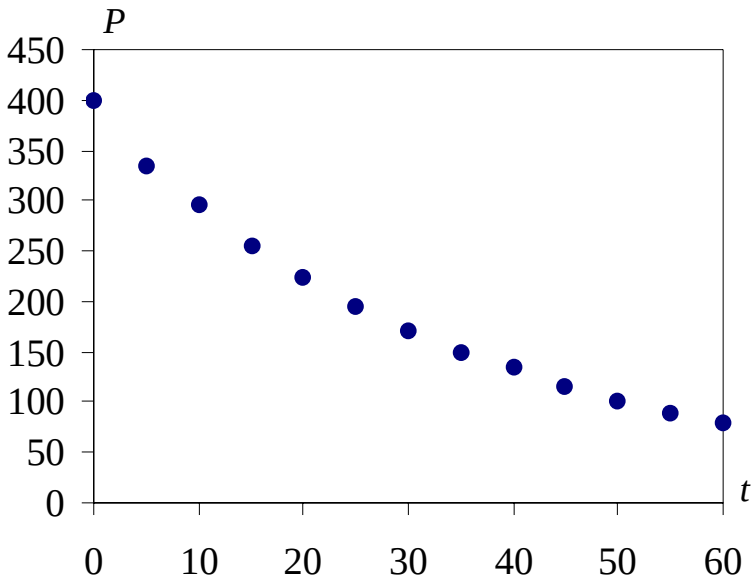


Starting pressure: $P(0) = 400$ kPa.
Every minute, P decreases by 4%.
That means every minute P is multiplied by 0.96.
Find $P(t)$ the pressure after t minutes.

$$P(t) = 400(0.96)^t$$



So let's go back to our data.



We want to see whether it has this "constant multiplier" form.

$$P(t) = 400r^{t}$$

time t (min)	pressure P (kPa)
0	400
5	335
10	295
15	255
20	225
25	195
30	170
35	150
40	135
45	115
50	100
55	90
60	80

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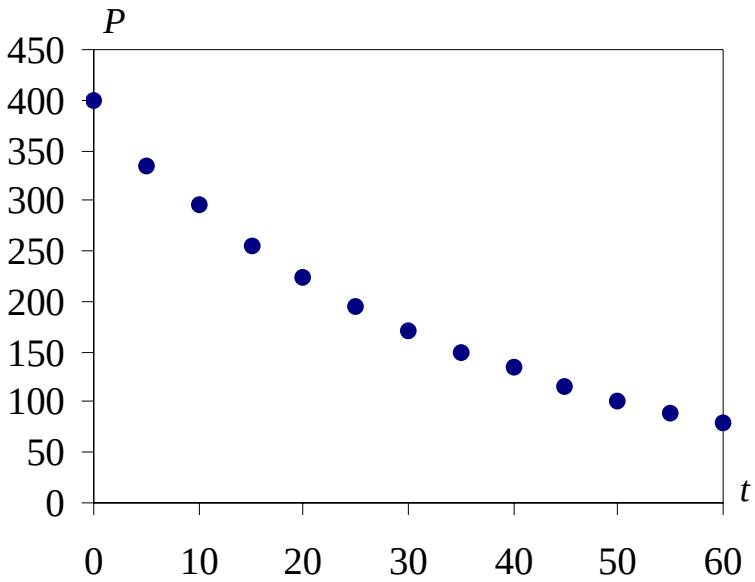

We want to see whether it has this "constant multiplier" form.

$$P(t) = 400r^t$$

Lots of ways to do this.

time t (min)	pressure P (kPa)
0	400
5	335
10	295
15	255
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35	150
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$$P(0) = 400r^0$$

$$P(1) = 400r^1$$

$$P(2) = 400r^2$$

$$P(3) = 400r^3$$

$$P(4) = 400r^4$$

$$\vdots$$

time t (min)	pressure P (kPa)
0	400
5	335
10	295
15	255
20	225
25	195
30	170
35	150
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$$P(3) = 400r^3$$

$$P(4) = 400r^4$$

$$\vdots$$

Work at the level of the index

time t (min)	pressure P (kPa)
0	400
5	335
10	295
15	255
20	225
25	195
30	170
35	150
40	135
45	115
50	100
55	90
60	80

$$P(t) = 400r^{t}$$

Let: $400 = 10^{a}$

$$r = 10^{b}$$

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5	335
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Let: $400 = 10^{a}$

$$r = 10^{b}$$

$$\begin{array}{lll} 400 & = 10^{a} & = 10^{a} \\ 400r & = 10^{a}(10^{b}) & = 10^{a+b} \\ 400r^{2} & = 10^{a}(10^{b})^{2} & = 10^{a+2b} \\ 400r^{3} & = 10^{a}(10^{b})^{3} & = 10^{a+3b} \\ 400r^{4} & = 10^{a}(10^{b})^{4} & = 10^{a+4b} \\ & \vdots & \end{array}$$

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50	100
55	90
60	80

The index increases additively

$$
\begin{array}{lll}
400 & = 10^{a} & = 10^{a} \\
400r & = 10^{a}(10^{b}) & = 10^{a+b} \\
400r^{2} & = 10^{a}(10^{b})^{2} & = 10^{a+2b} \\
400r^{3} & = 10^{a}(10^{b})^{3} & = 10^{a+3b} \\
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 & \vdots &
\end{array}
$$

time t (min)	pressure P (kPa)
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The index increases additively

And plots as a straight line

$$
\begin{array}{lll}
400 & = 10^{a} & = 10^{a} \\
400r & = 10^{a}(10^{b}) & = 10^{a+b} \\
400r^{2} & = 10^{a}(10^{b})^{2} & = 10^{a+2b} \\
400r^{3} & = 10^{a}(10^{b})^{3} & = 10^{a+3b} \\
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35	150
40	135
45	115
50	100
55	90
60	80

So let's write the data as powers of 10

And see whether we get a straight line.

t	P	index
0	$400 = 10^{2.60}$	2.60
5	$335 = 10^{2.52}$	2.53
10	$295 = 10^{2.47}$	2.47
15	$255 = 10^{2.41}$	2.41
20	$225 = 10^{2.35}$	2.35
25	$195 = 10^{2.29}$	2.29
30	$170 = 10^{2.23}$	2.23
35	$150 = 10^{2.18}$	2.18
40	$135 = 10^{2.13}$	2.13
45	$115 = 10^{2.06}$	2.06
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55	$90 = 10^{1.95}$	1.95
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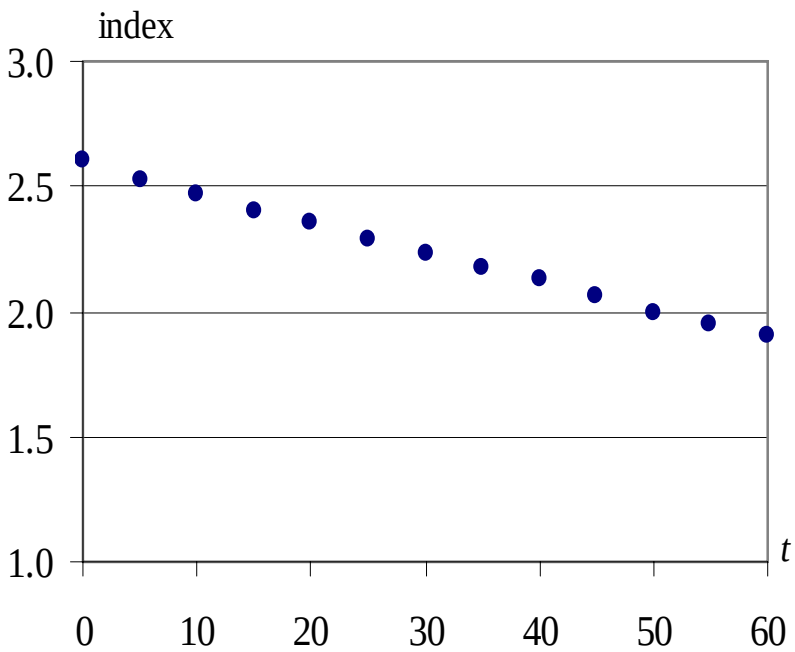

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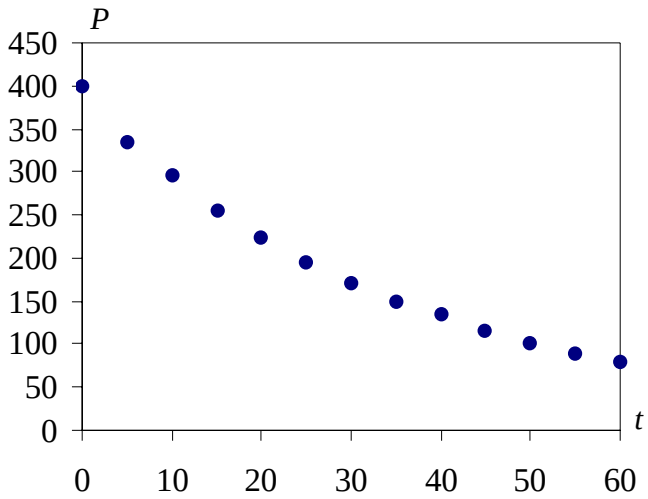

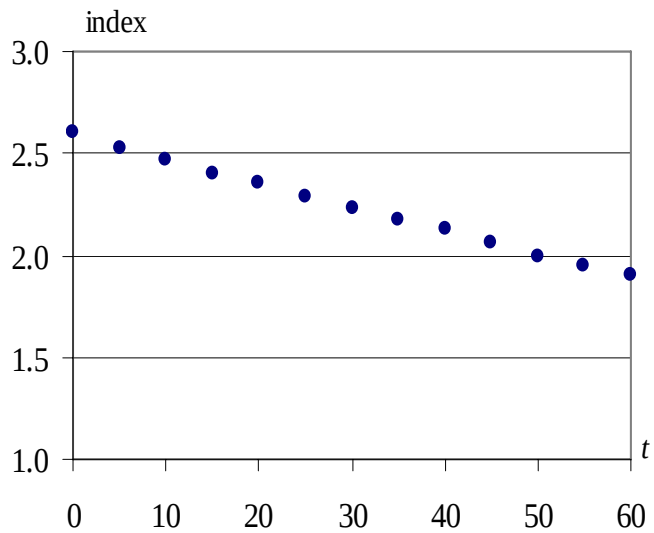

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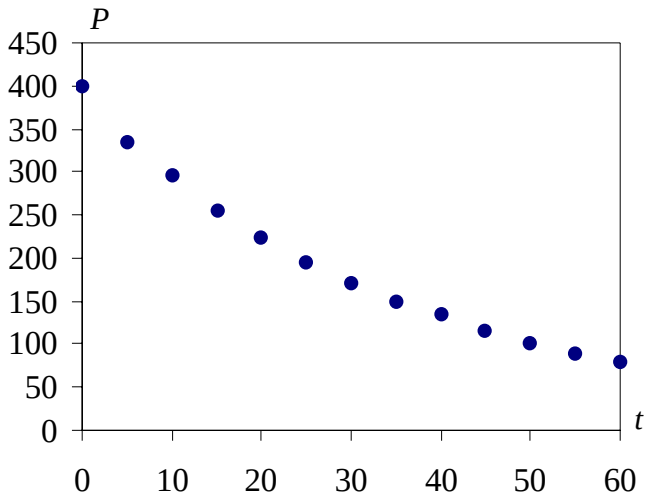

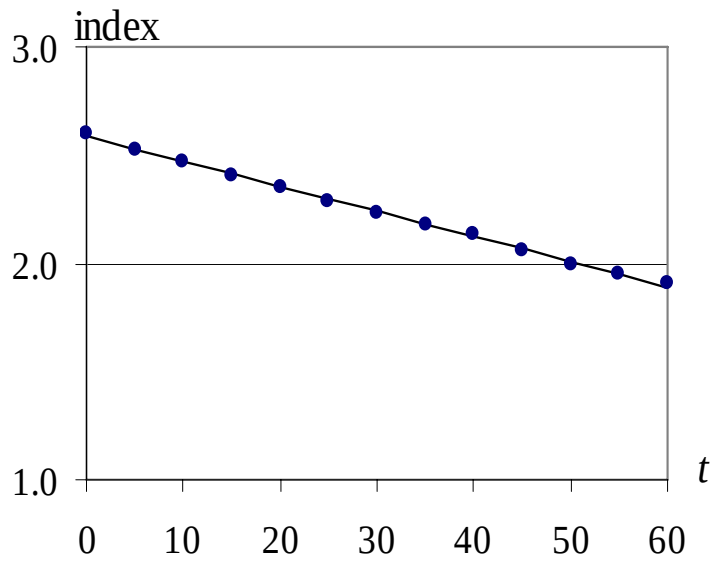

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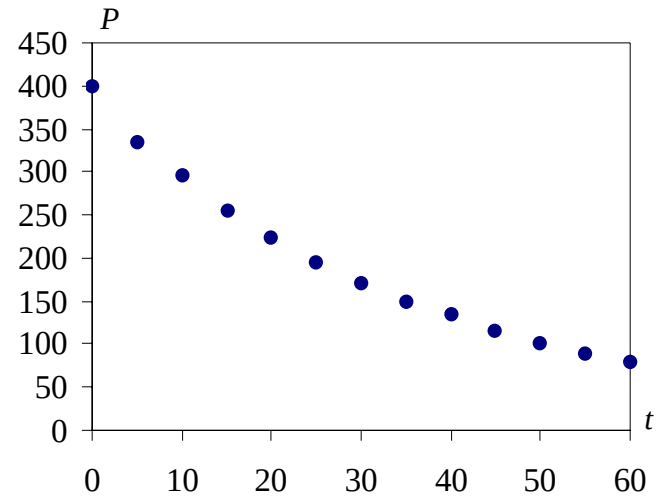
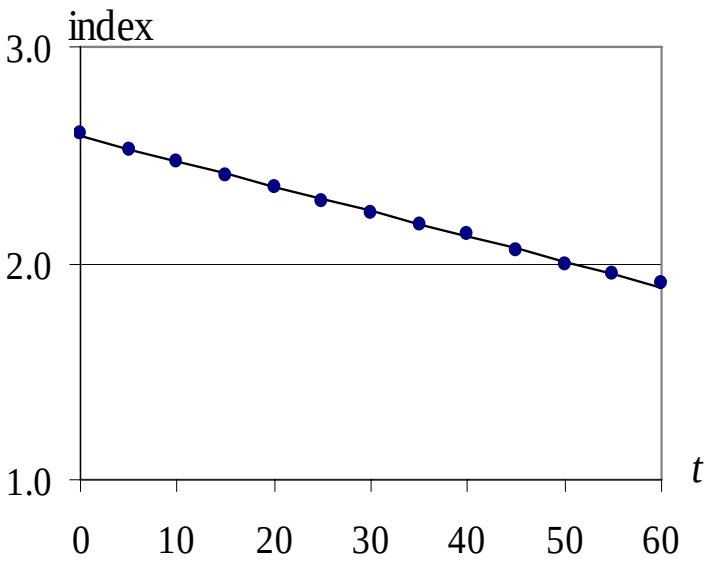
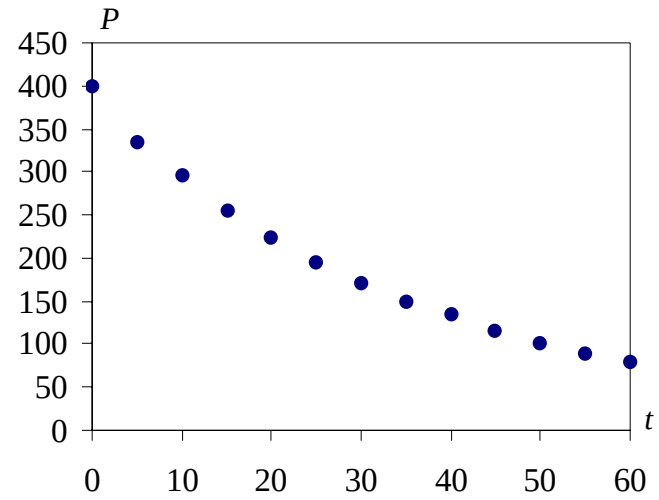
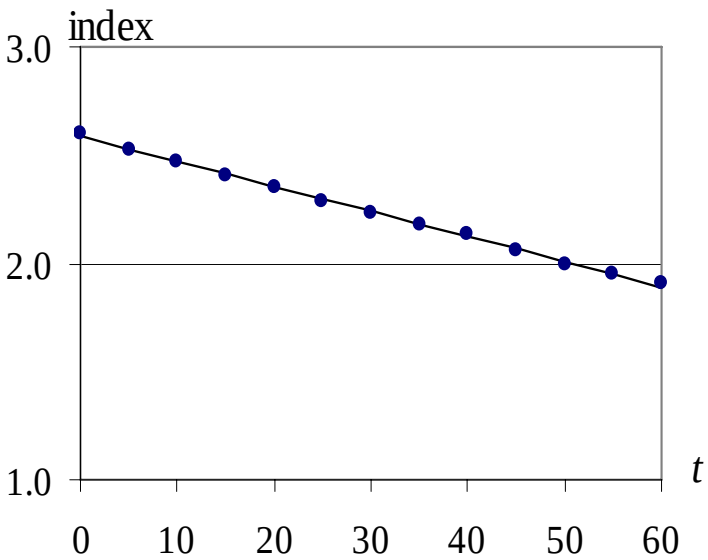




$$\text{index} = 2.5854 - 0.01156t$$



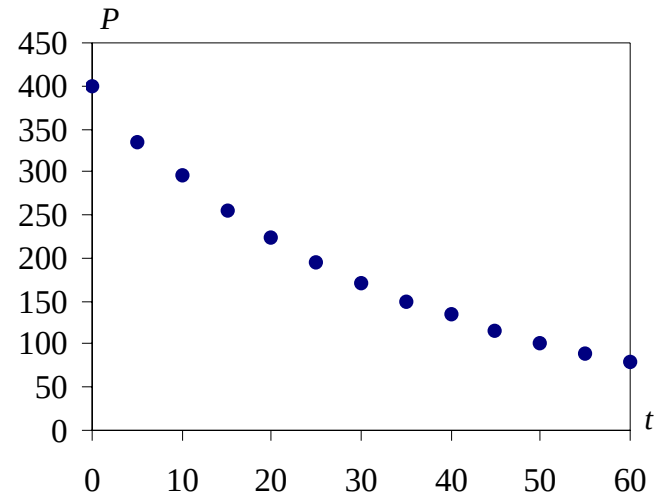


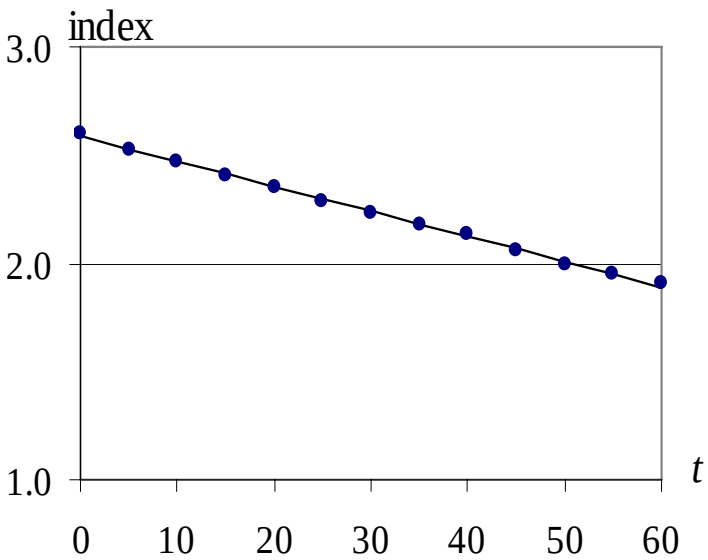
$$\text{index} = 2.5854 - 0.01156t$$

$$P = 10^{index} = 10^{25854 - 0.01156t}$$

$$= 10^{25854} 10^{-0.01156t}$$

$$P = 385(0.974)^t$$





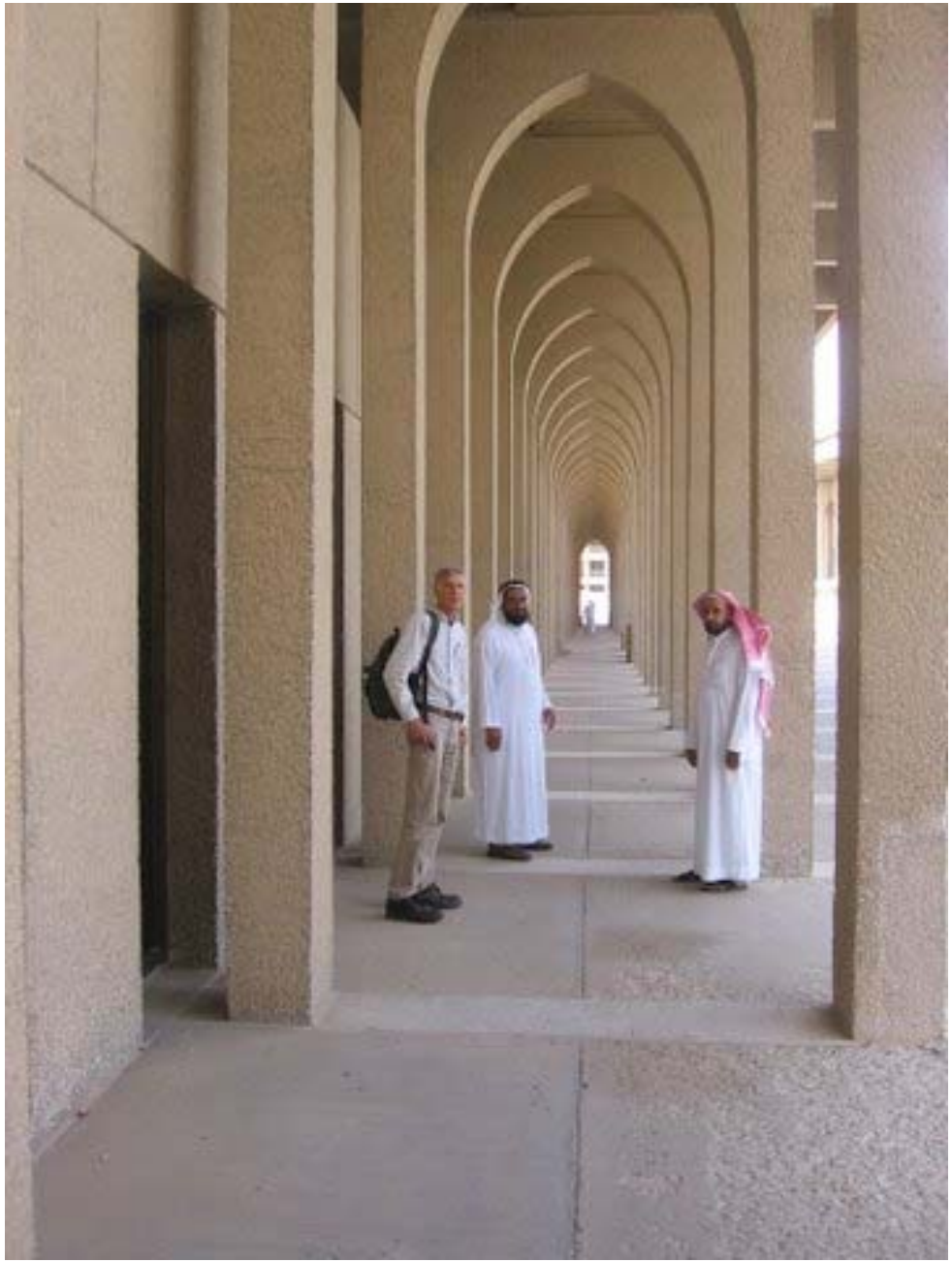
$$\text{index} = 2.5854 - 0.01156t$$

$$P = 10^{index} = 10^{25854 - 0.01156t}$$

$$= 10^{25854} 10^{-0.01156t}$$

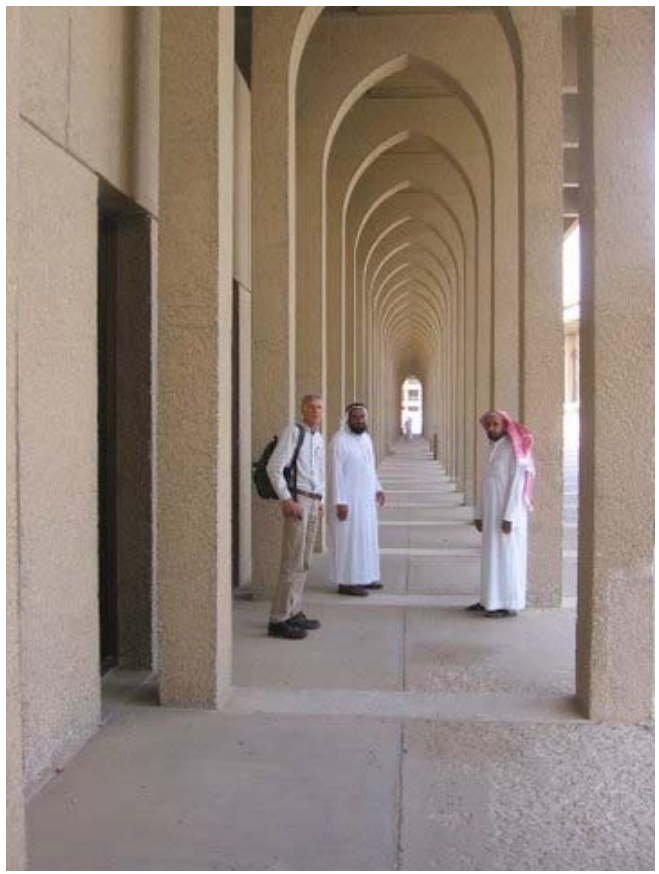
$$P = 385(0.974)^t$$

Tire pressure drops by 2.6% per minute.

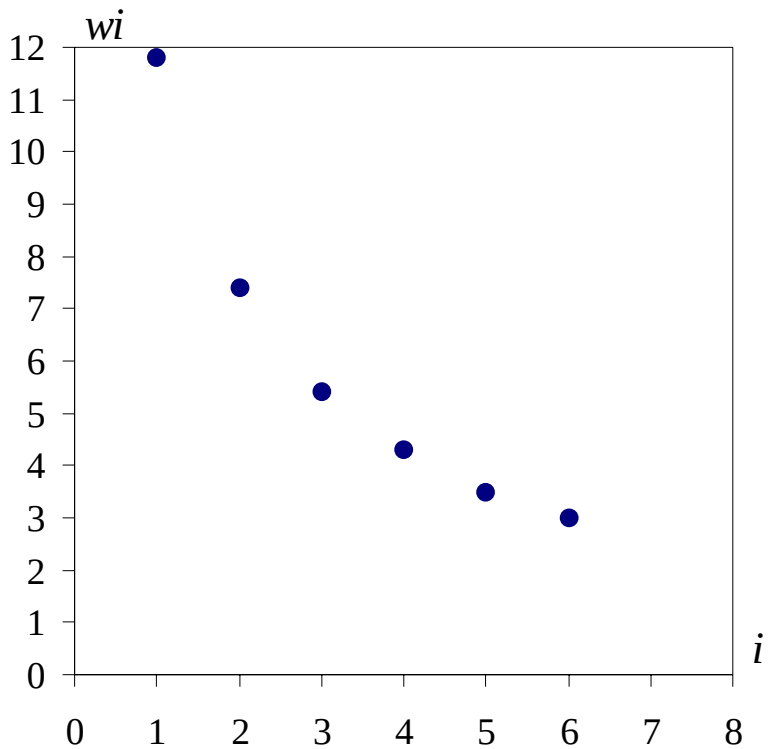


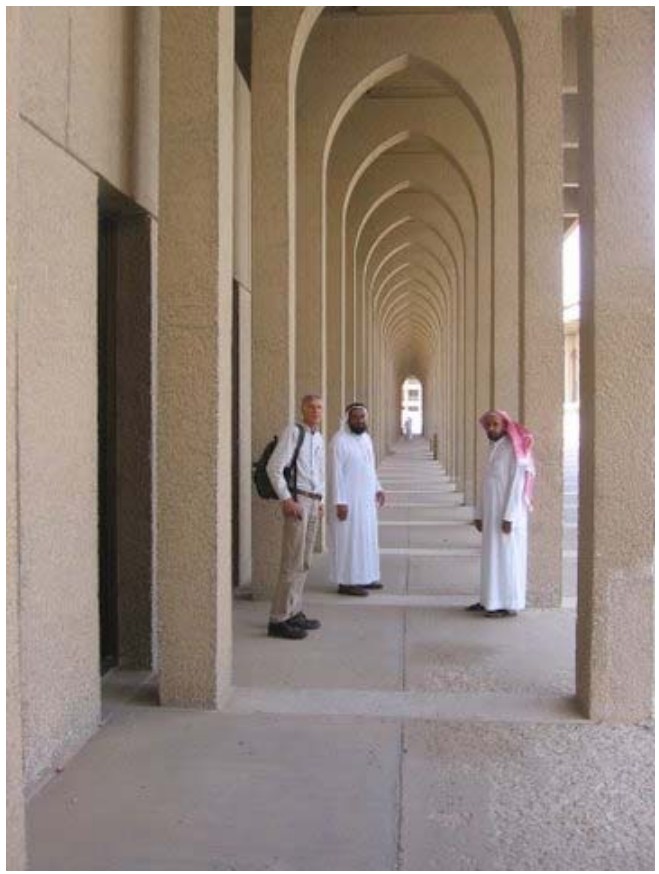
Let w_i be the width of the i th arch
As seen on the page.

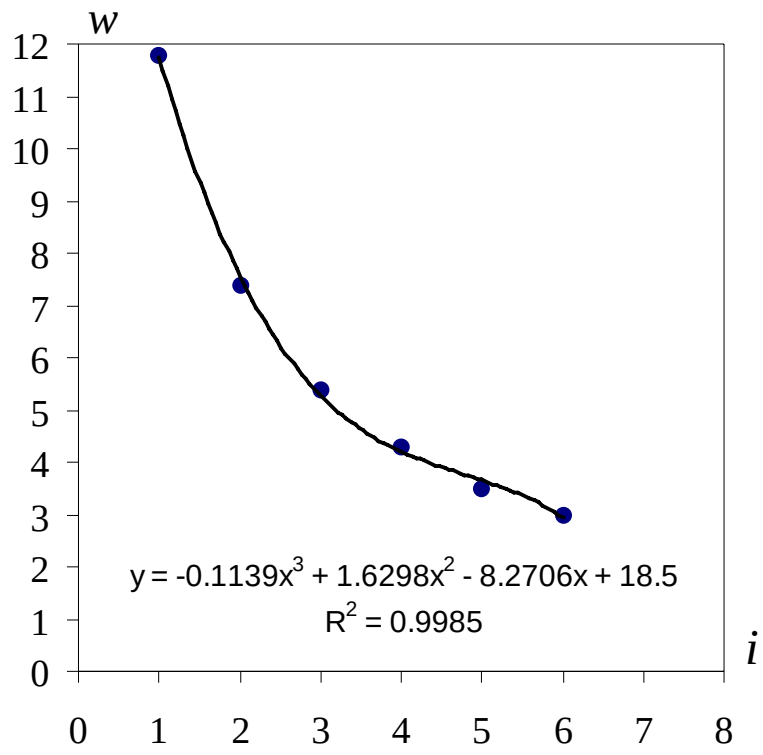
What kind of function is w_i ?



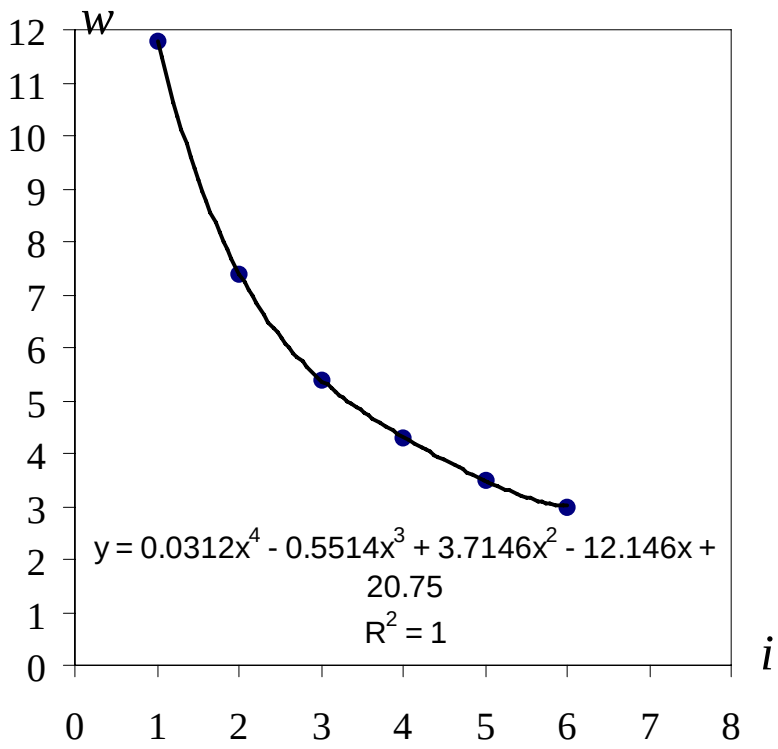
i	w_i
1	11.8
2	7.4
3	5.4
4	4.3
5	3.5
6	3.0



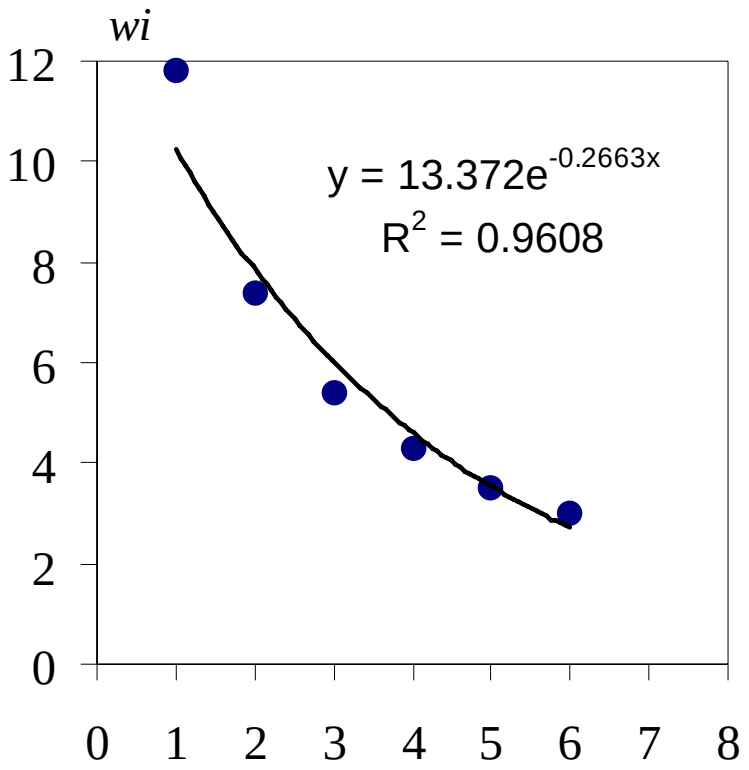




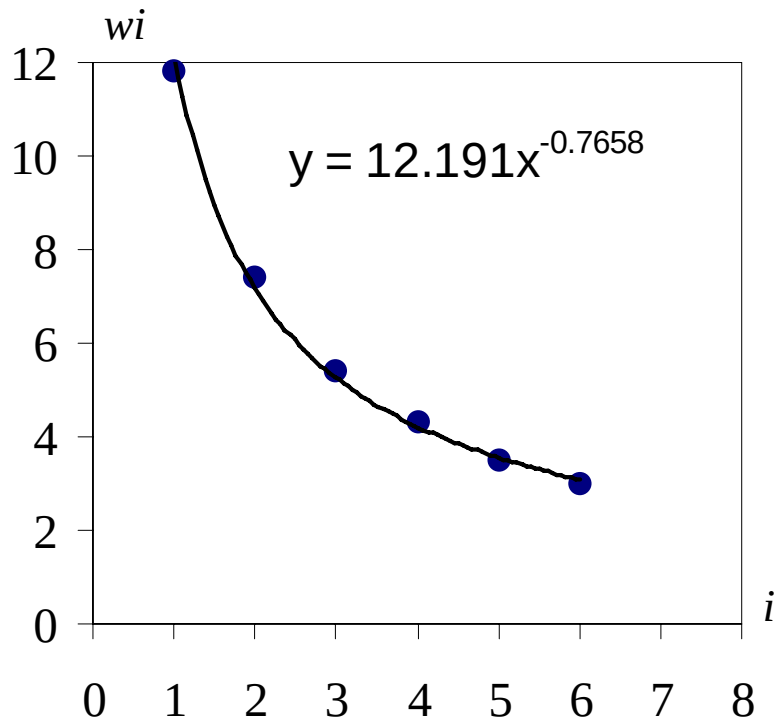
cubic curve



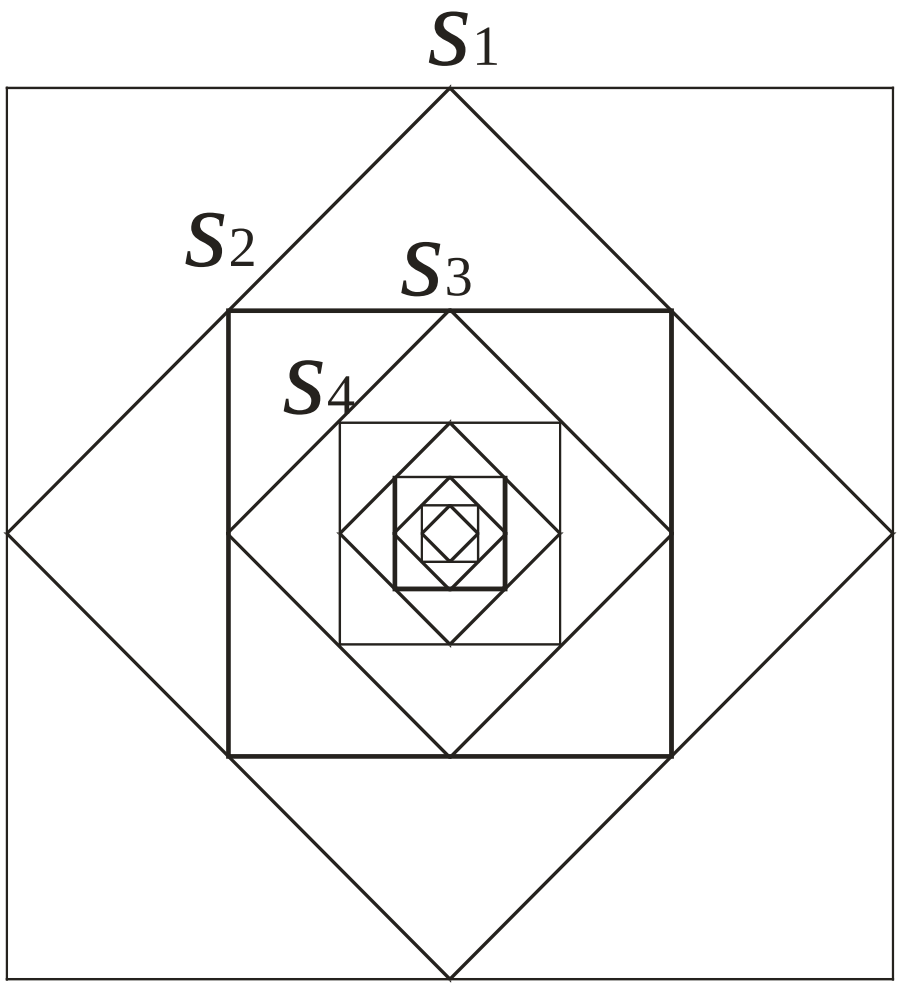
quartic curve

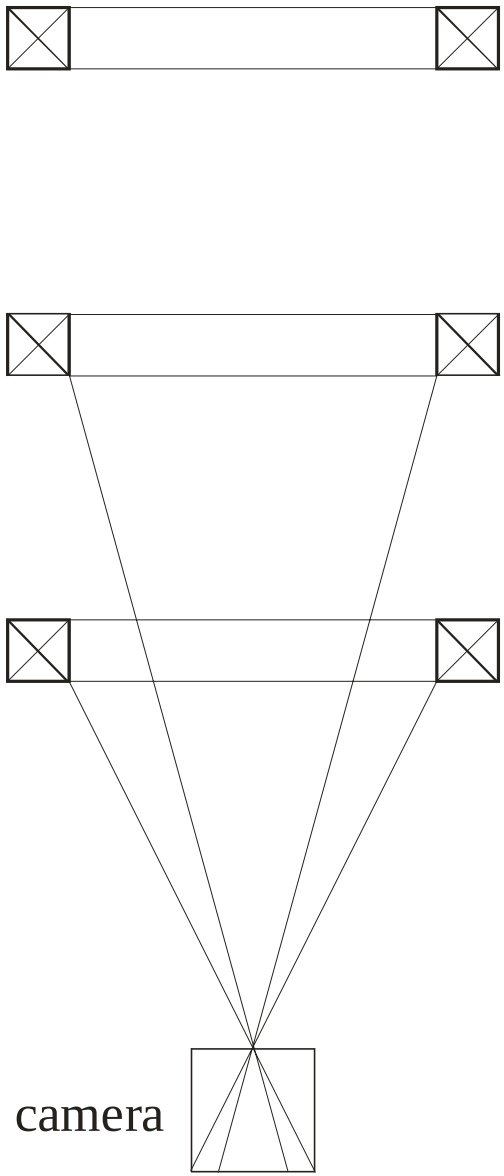


Exponential curve



Power function

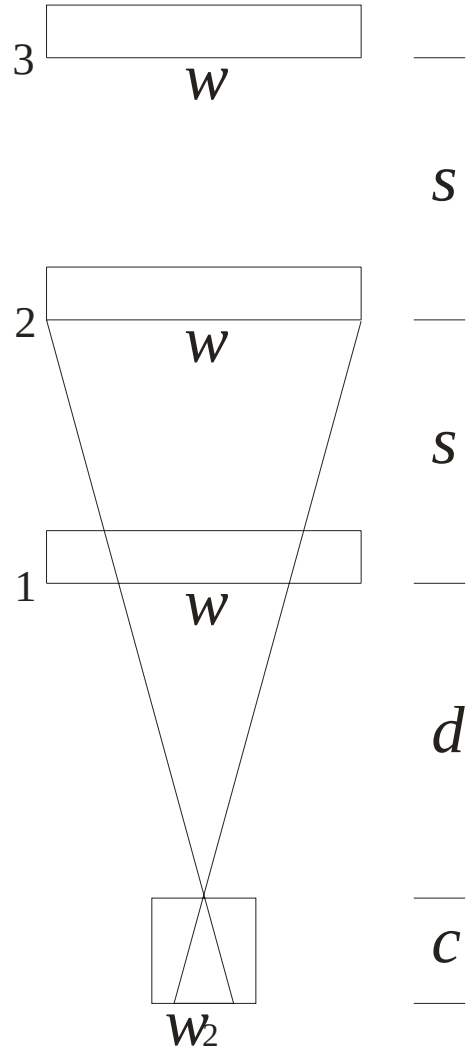

S_1
S_2
S_3
S_4

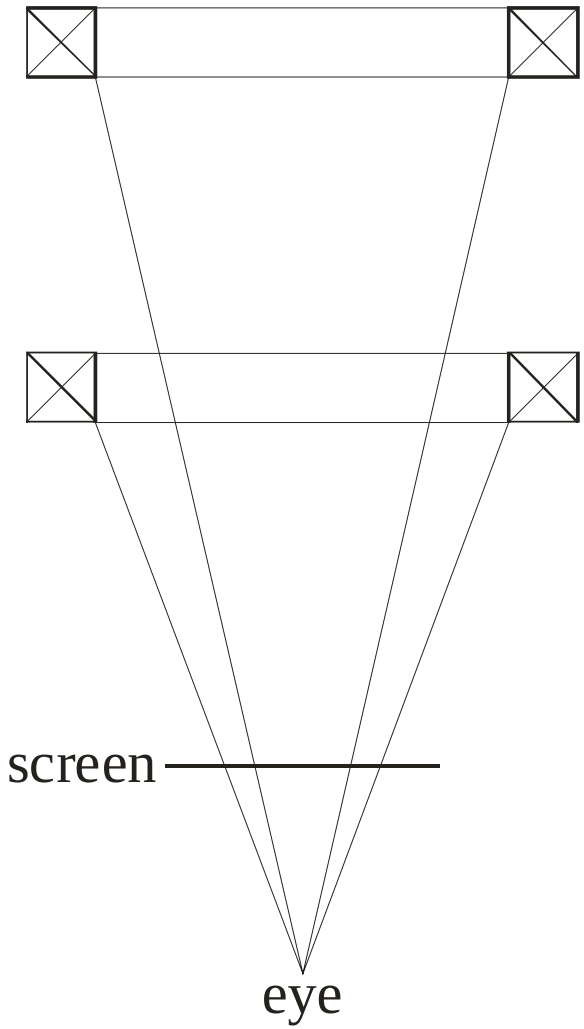

camera

$$\frac{w_2}{c} = \frac{w}{d+s}$$

$$\frac{w_i}{c} = \frac{w}{d+(i-1)s}$$

$$w_i = \frac{cw}{d-s+is}$$




screen
eye

U_1	1
U_2	1
U_3	2
U_4	3
U_5	5
U_6	8
U_7	13
U_8	21

U_9	34
U_{10}	55
U_{11}	89
U_{12}	144
U_{13}	233
U_{14}	377
U_{15}	610
U_{16}	987

U_1	1
U_2	1
U_3	2
U_4	3
U_5	5
U_6	8
U_7	13
U_8	21

25

64

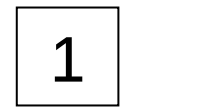
U_9	34
U_{10}	55
U_{11}	89
U_{12}	144
U_{13}	233
U_{14}	377
U_{15}	610
U_{16}	987

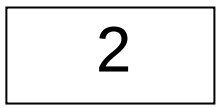
$$5^2 + 8^2 = 89$$

$$u_5{}^2 + u_6{}^2 = u_{11}$$

“Trains” model of the Fibonacci sequence

Build all possible trains using cars of length 1 and cars of length 2.





Let t_n be the number of trains of length n

$t_3 = 3$

111
12
21

$t_4 = 5$

1111
112
121
211
22

$t_5 = 8$

11111
1112
1121
1211
2111
122
212
221

The t_n are the Fibonacci numbers

Except that $t_4 = u_5$

In general $t_n = u_{n+1}$

$$5^2 + 8^2 = 89$$

$$u_5{}^2 + u_6{}^2 = u_{11}$$

$$t_4{}^2 + t_5{}^2 = t_{10}$$

Can you see why this might be true?